MACHINE DESIGN

An Integrated Approach

Fourth Edition

Robert L. Norton

MACHINE DESIGN

An Integrated Approach

Fourth Edition

Robert L. Norton

Worcester Polytechnic Institute

Worcester, Massachusetts

Prentice Hall

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto
Delhi Mexico City Sao Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

ABOUT THE AUTHOR

Robert L. Norton earned undergraduate degrees in both mechanical engineering and industrial technology at Northeastern University and an MS in engineering design at Tufts University. He is a registered professional engineer in Massachusetts. He has extensive industrial experience in engineering design and manufacturing and many years' experience teaching mechanical engineering, engineering design, computer science, and related subjects at Northeastern University, Tufts University, and Worcester Polytechnic Institute.

At Polaroid Corporation for 10 years, he designed cameras, related mechanisms, and high-speed automated machinery. He spent three years at Jet Spray Cooler Inc., designing food-handling machinery and products. For five years he helped develop artificial-heart and noninvasive assisted-circulation (counterpulsation) devices at the Tufts New England Medical Center and Boston City Hospital. Since leaving industry to join academia, he has continued as an independent consultant on engineering projects ranging from disposable medical products to high-speed production machinery. He holds 13 U.S. patents.

Norton has been on the faculty of Worcester Polytechnic Institute since 1981 and is currently the Milton P. Higgins II Distinguished Professor of Mechanical Engineering, Russell P. Searle Distinguished Instructor, Head of the Design Group in that department, and the Director of the Gillette Project Center at WPI. He teaches undergraduate and graduate courses in mechanical engineering with emphasis on design, kinematics, vibrations, and dynamics of machinery.

He is the author of numerous technical papers and journal articles covering kinematics, dynamics of machinery, cam design and manufacturing, computers in education, and engineering education and of the texts *Design of Machinery*, *Machine Design: An Integrated Approach* and the *Cam Design and Manufacturing Handbook*. He is a Fellow of the American Society of Mechanical Engineers and a member of the Society of Automotive Engineers. But, since his main interest is in teaching, he is most proud of the fact that, in 2007, he was chosen as *U. S. Professor of the Year* for the State of Massachusetts by the *Council for the Advancement and Support of Education (CASE)* and the *Carnegie Foundation for the Advancement of Teaching*, who jointly present the only national awards for teaching excellence given in the United States of America.

This book is dedicated to:

Donald N. Zwiep

Provost, Department Head, and Professor Emeritus
Worcester Polytechnic Institute

A gentleman and a leader,
without whose faith and foresight,
this book would never have been written.

Contents

CHAPTER 2 MATERIALS AND PROCESSES 29

Part II Machine Design 521

Preface

Introduction

This text is intended for the *Design of Machine Elements* courses typically given in the junior year of most mechanical engineering curricula. The usual prerequisites are a first course in *Statics and Dynamics*, and one in *Strength of Materials*. The purpose of this book is to present the subject matter in an up-to-date manner with a strong design emphasis. The level is aimed at junior-senior mechanical engineering students. A primary goal was to write a text that is very easy to read and that students will enjoy reading despite the inherent dryness of the subject matter.

This textbook is designed to be an improvement over others currently available and to provide methods and techniques that take full advantage of computer-aided analysis. It emphasizes design and synthesis as well as analysis. Example problems, case studies, and solution techniques are spelled out in detail and are self-contained. All the illustrations are done in two colors. Short problems are provided in each chapter and, where appropriate, longer unstructured design-project assignments are given.

The book is independent of any particular computer program. Computer files for the solution of all the examples and case studies written in several different languages (Mathcad, MATLAB, Excel, and TK Solver) are provided on the CD-ROM. Several other programs written by the author are also provided as executable files. These include a Mohr's circle generator (MOHR.exe), dynamic surface stress calculator (CONTACT.exe), matrix solver (MATRIX.exe) and several linkage and cam design programs. An index of the CD-ROM's content is on the CD.

While this book attempts to be thorough and complete on the engineering-mechanics topics of failure theory and analysis, it also emphasizes the synthesis and design aspects of the subject to a greater degree than most other texts in print on this subject. It points out the commonality of the analytical approaches needed to design a wide variety of elements and emphasizes the use of computer-aided engineering as an approach to the design and analysis of these classes of problems. The author's approach to this course is based on 50 years of practical experience in mechanical engineering design, both in industry and as a consultant. He has taught mechanical engineering design at the university level for 30 of those years as well.

What's New in the Fourth Edition?

- A new chapter on the design of weldments presents the latest data and methods on this topic.
- The chapter on finite element analysis (FEA) has been moved from Chapter 16 to Chapter 8 and augmented with additional FEA solutions for case studies that are developed in earlier chapters.
- Solidworks models with FEA solutions to several of the case studies are provided on the CD-ROM.
- Solidworks models of many assigned problems' geometry are provided on the CD-ROM to expedite FEA solutions of those problems at the instructor's option.
- A new technique for the computation of bolted-joint stiffness is presented in Chapter 15 on Fasteners.
- Over 150 problems are added or revised with an emphasis on SI units.

Philosophy

This is often the first course that mechanical engineering students see that presents them with design challenges rather than set-piece problems. Nevertheless, the type of design addressed in this course is that of *detailed design,* which is only one part of the entire design-process spectrum. In detailed design, the general concept, application, and even general shape of the required device are typically known at the outset. We are not trying to invent a new device so much as define the shape, size, and material of a particular machine element such that it will not fail under the loading and environmental conditions expected in service.

The traditional approach to the teaching of the *Elements* course has been to emphasize the design of individual machine parts, or elements, such as gears, springs, shafts, etc. One criticism that is sometimes directed at the *Elements* course (or textbook) is that it can easily become a "cookbook" collection of disparate topics that does not prepare the student to solve other types of problems not found in the recipes presented. There is a risk of this happening. It is relatively easy for the instructor (or author) to allow the course (or text) to degenerate into the mode "Well it's Tuesday, let's design springs—on Friday, we'll do gears." If this happens, it may do the student a disservice because it doesn't necessarily develop a fundamental understanding of the practical application of the underlying theories to design problems.

However, many of the machine elements typically addressed in this course provide superb examples of the underlying theory. If viewed in that light, and if presented in a general context, they can be an excellent vehicle for the development of student understanding of complex and important engineering theories. For example, the topic of preloaded bolts is a perfect vehicle to introduce the concept of prestressing used as a foil against fatigue loading. The student may never be called upon in practice to design a preloaded bolt, but he or she may well utilize the understanding of prestressing gained from the experience. The design of helical gears to withstand time-varying loads provides an excellent vehicle to develop the student's understanding of combined stresses, Hertzian stresses, and fatigue failure. Thus the *elements* approach is a valid and defensible one as long as the approach taken in the text is sufficiently global. That is, it should not be allowed to degenerate into a collection of apparently unrelated exercises, but rather provide an integrated approach.

Another area in which the author has found existing texts (and *Machine Elements* courses) to be deficient is the lack of connection made between the dynamics of a system and the stress analysis of that system. Typically, these texts present their machine elements with (magically) predefined forces on them. The student is then shown how to determine the stresses and deflections caused by those forces. In real machine design, the forces are not always predefined and can, in large part, be due to the accelerations of the masses of the moving parts. However, the masses cannot be accurately determined until the geometry is defined and a stress analysis done to determine the strength of the assumed part. Thus an impasse exists that is broken only by iteration, i.e., assume a part geometry and define its geometric and mass properties, calculate the dynamic loads due in part to the material and geometry of the part. Then calculate the stresses and deflections resulting from those forces, find out it fails, redesign, and repeat.

An Integrated Approach

The text is divided into two parts. The first part presents the fundamentals of stress, strain, deflection, materials properties, failure theories, fatigue phenomena, fracture mechanics, FEA, etc. These theoretical aspects are presented in similar fashion to other texts. The second part presents treatments of specific, common design elements used as examples of applications of the theory but also attempts to avoid presenting a string of disparate topics in favor of an integrated approach that ties the various topics together via *case studies*.

Most *Elements* texts contain many more topics and more content than can possibly be covered in a one-semester course. Before writing the first edition of this book, a questionnaire was sent to 200 U.S. university instructors of the *Elements* course to solicit their opinions on the relative importance and desirability of the typical set of topics in an *Elements* text. With each revision to second , third, and fourth editions, users were again surveyed to determine what should be changed or added. The responses were analyzed and used to influence the structure and content of this book in all editions. One of the strongest desires originally expressed by the respondents was for *case studies* that present realistic design problems.

We have attempted to accomplish this goal by structuring the text around a series of ten case studies. These case studies present different aspects of the same design problem in successive chapters, for example, defining the static or dynamic loads on the device in Chapter 3, calculating the stresses due to the static loads in Chapter 4, and applying the appropriate failure theory to determine its safety factor in Chapter 5. Later chapters present more complex case studies, with more design content. The case study in Chapter 6 on fatigue design is one such example a real problem taken from the author's consulting practice. Chapter 8 presents FEA analyses of several of these case studies and compares those results to the classical solutions done in prior chapters.

The case studies provide a series of machine design projects throughout the book that contain various combinations of the elements normally dealt with in this type of text. The assemblies contain some collection of elements such as links subjected to combined axial and bending loads, column members, shafts in combined bending and torsion, gearsets under alternating loads, return springs, fasteners under fatigue loading, rolling element bearings, etc. This integrated approach has several advantages. It presents the student with a generic design problem in context rather than as a set of disparate, unrelated entities. The student can then see the interrelationships and the rationales for the design decisions that affect the individual elements. These more comprehensive case studies are in Part II of the text. The case studies in Part I are more limited in scope and directed to the engineering mechanics topics of the chapter. In addition to the case studies, each chapter has a selection of worked-out examples to reinforce particular topics.

Chapter 9, Design Case Studies, is devoted to the setup of three design case studies that are used in the following chapters to reinforce the concepts behind the design and analysis of shafts, springs, gears, fasteners, etc. Not all aspects of these design case studies are addressed as worked-out examples since another purpose is to provide material for student-project assignments. The author has used these case study topics as multi-week or term-long project assignments for groups or individual students with good success. Assigning open-ended project assignments serves to reinforce the design and analysis aspects of the course much better than set-piece homework assignments.

Problem Sets

Most of the 790 problem sets (590, or 75%) are independent within a chapter, responding to requests by users of the first edition to decouple them. The other 25% of the problem sets are still built upon in succeeding chapters. These linked problems have the same dash number in each chapter and their problem number is **boldface** to indicate their commonality among chapters. For example, Problem 3-4 asks for a static force analysis of a trailer hitch; Problem 4-4 requests a stress analysis of the same hitch based on the forces calculated in Problem 3-4; Problem 5-4 asks for the static safety factor for the hitch using the stresses calculated in Problem 4-4; Problem 6-4 requests a fatigue-failure analysis of the same hitch, and Problem 7-4 requires a surface stress analysis. The same trailer hitch is used as an FEA case study in Chapter 8. Thus, the complexity of the underlying design problem is unfolded as new topics are introduced. An instructor who wishes to use this approach can assign problems with the same dash number in succeeding chapters. If one does not want to assign an earlier problem on which a later one is based, the solution manual data from the earlier problem can be provided to the students. Instructors who do not like interlinked problems can avoid them entirely and select from the 590 problems with nonbold problem numbers that are independent within their chapters.

Text Arrangement

Chapter 1 provides an introduction to the design process, problem formulation, safety factors, and units. Material properties are reviewed in Chapter 2 since even the student who has had a first course in material science or metallurgy typically has but a superficial understanding of the wide spectrum of engineering material properties needed for machine design. Chapter 3 presents a review of static and dynamic loading analysis, including beam, vibration, and impact loading, and sets up a series of case studies that are used in later chapters to illustrate the stress and deflection analysis topics with some continuity.

The *Design of Machine Elements* course, at its core, is really an intermediate-level, applied stress-analysis course. Accordingly, a review of the fundamentals of stress and deflection analysis is presented in Chapter 4. Static failure theories are presented in detail in Chapter 5 since the students have typically not yet fully digested these concepts from their first stress-analysis course. Fracture-mechanics analysis for static loads is also introduced.

The *Elements* course is typically the student's first exposure to fatigue analysis since most introductory stress-analysis courses deal only with statically loaded problems. Accordingly, fatigue-failure theory is presented at length in Chapter 6 with the emphasis on stress-life approaches to high-cycle fatigue design, which is commonly used in the design of rotating machinery. Fracture-mechanics theory is further discussed with regard to crack propagation under cyclic loading. Strain-based methods for low-cycle fatigue analysis are not presented but their application and purpose are introduced to the reader and bibliographic references are provided for further study. Residual stresses are also addressed. Chapter 7 presents a thorough discussion of the phenomena of wear mechanisms, surface contact stresses, and surface fatigue.

Chapter 8 provides an introduction to Finite Element Analysis (FEA). Many instructors are using the machine elements course to introduce students to FEA as well as to instruct them in the techniques of machine design. The material presented in Chapter 8 is not intended as a substitute for education in FEA theory. That material is available in many other textbooks devoted to that subject and the student is urged to become familiar with FEA theory through coursework or self-study. Instead Chapter 8 presents proper techniques for the application of FEA to practical machine design problems. Issues of element selection, mesh refinement, and the definition of proper boundary conditions are developed in some detail. These issues are not usually addressed in books on FEA theory. Many engineers in training today will, in their professional practice, use CAD solid modeling software and commercial finite element analysis code. It is important that they have some knowledge of the limitations and proper application of those tools. This chapter can be taken up earlier in the course if desired, especially if the students are expected to use FEA to solve assigned tasks. It is relatively independent of the other chapters. Many of various chapters' problem assignments have Solidworks models of their geometry provided on the CD-ROM.

These eight chapters comprise Part I of the text and lay the analytical foundation needed for design of machine elements. They are arranged to be taken up in the order presented and build upon each other with the exception of Chapter 8 on FEA.

Part II of the text presents the design of machine elements in context as parts of a whole machine. The chapters in Part II are essentially independent of one another and can be taken (or skipped) in any order that the instructor desires (except that Chapter 12 on spur gears should be studied before Chapter 13 on helical, bevel, and worm gears). It is unlikely that all topics in the book can be covered in a one-term or one semester course. Uncovered chapters will still serve as a reference for engineers in their professional practice.

Chapter 9 presents a set of design case studies to be used as assignments and as example case studies in the following chapters and also provides a set of suggested design project assignments in addition to the detailed case studies as described above. Chapter 10 investigates shaft design using the fatigue-analysis techniques developed in Chapter 6. Chapter 11 discusses fluid-film and rolling-element bearing theory and application using the theory developed in Chapter 7. Chapter 12 gives a thorough introduction to the kinematics, design and stress analysis of spur gears using the latest AGMA recommended procedures. Chapter 13 extends gear design to helical, bevel, and worm gearing. Chapter 14 covers spring design including helical compression, extension and torsion springs, as well as a thorough treatment of Belleville springs. Chapter 15 deals with screws and fasteners including power screws and preloaded fasteners. Chapter 16 presents an up-to-date treatment of the design of weldments for both static and dynamic loading. Chapter 17 presents an introduction to the design and specification of disk and drum clutches and brakes. The appendices contain material-strength data, beam tables, and stress-concentration factors, as well as answers to selected problems.

Supplements

A **Solutions Manual** is available to instructors from the publisher and **PowerPoint slides** of all figures and tables in the text are available on the publisher's website (password protected) at:

http://www.pearsonhighered.com/

To download these resources, choose the **Instructor Support** tab to register as an instructor and follow instructions on the site to obtain the resources provided. Mathcad files for all the problem solutions are available with the solutions manual. This computerized approach to problem solutions has significant advantages to the instructor who can easily change any assigned problem's data and instantly solve it. Thus an essentially infinite supply of problem sets is available, going far beyond those defined in the text. The instructor also can easily prepare and solve exam problems by changing data in the supplied files.

Anyone may download supplemental information about the author's course organization and operation (syllabi, project assignments, etc.) from the author's university web site at:

http://www.me.wpi.edu/People/Norton/design.html

As errata are discovered they will be posted on the author's personal website at:

http://www.designofmachinery.com/MD/errata.html

Professors who adopt the book may register at the author's personal website to obtain additional information relevant to the subject and the text and to download updated software (password protected). Go to:

http://www.designofmachinery.com/registered/professor.html

Anyone who purchases the book may register at the author's personal website to request updated software for the current edition (password protected). Go to:

http://www.designofmachinery.com/registered/student.html

Acknowledgments

The author expresses his sincere appreciation to all those who reviewed the first edition of the text in various stages of development including Professors J. E. Beard, Michigan Tech; J. M. Henderson, U. California, Davis; L. R. Koval, U. Missouri, Rolla; S. N. Kramer, U. Toledo; L. D. Mitchell, Virginia Polytechnic; G. R. Pennock, Purdue; D. A. Wilson, Tennessee Tech; Mr. John Lothrop; and Professor J. Ari-Gur, Western Michigan University, who also taught from a class-test version of the book. Robert Herrmann (WPI-ME ‘94) provided some problems and Charles Gillis (WPI-ME ‘96) solved most of the problem sets for the first edition.

Professors John R. Steffen of Valparaiso University, R. Jay Conant of Montana State, Norman E. Dowling of Virginia Polytechnic, and Francis E. Kennedy of Dartmouth made many useful suggestions for improvement and caught many errors. Special thanks go to Professor Hartley T. Grandin of WPI, who provided much encouragement and many good suggestions and ideas throughout the book’s gestation, and also taught from various class-test versions.

Three former and the current Prentice Hall editors deserve special mention for their efforts in developing this book: Doug Humphrey, who wouldn’t take no for an answer in persuading me to write it, Bill Stenquist, who usually said yes to my requests and expertly shepherded the book through to completion in its first edition, and Eric Svendsen who helped get the third edition into print and added value to the book. Tacy Quinn’s support has helped marshall the fourth edition into print.

Since the book’s first printing in 1995, several users have kindly pointed out errors and suggested improvements. My thanks go to Professors R. Boudreau of U. Moncton, Canada, V. Glozman of Cal Poly Pomona, John Steele of Colorado School of Mines, Burford J. Furman of San Jose State University, and Michael Ward of California State University, Chico.

Several other faculty have been kind enough to point out errors and offer constructive criticisms and suggestions for improvement in the later editions. Notable among these are: Professors Cosme Furlong of Worcester Polytechnic Institute, Joseph Rencis of University of Arkansas, Annie Ross of Universite de Moncton, Andrew Ruina of Cornell University, Douglas Walcerz of York College, and Thomas Dresner of Mountain City, CA.

Dr. Duane Miller of Lincoln Electric Company provided invaluable help with Chapter 16 on weldments and reviewed several drafts. Professor Stephen Covey of St. Cloud State University, and engineers Gregory Aviza and Charles Gillis of P&G Gillette also provided valuable feedback on the weldment chapter. Professor Robert Cornwell of Seattle University reviewed the discussion in Chapter 15 of his new method for the calculation of bolted joint stiffness and his method for computing stress concentration in rectangular wire springs discussed in Chapter 14.

Professors Fabio Marcelo Peña Bustos of Universidad Autónoma de Manizales, Caldas, Colombia, and Juan L. Balsevich-Prieto of Universidad Católica Nuestra Señora de la Asunción, Asunción, Paraguay, were kind enough to point out errata in the Spanish translation.

Special thanks are due to William Jolley of The Gillette Company who created the FEA models for the examples and reviewed Chapter 8, and to Edwin Ryan, retired Vice President of Engineering at Gillette, who provided invaluable support. Donald A. Jacques of the UTC Fuel Cells division of the United Technologies Company also reviewed Chapter 8 on Finite Element Analysis and made many useful suggestions. Professor Eben C. Cobb of Worcester Polytechnic Institute and his student Thomas Watson created the Solidworks models of many problem assignments and case studies and solved the FEA for the case studies that are on the CD-ROM.

Thanks are due several people who responded to surveys for the fourth edition and made many good suggestions: Kenneth R. Halliday of Ohio State University, Mohamed B. Trabia of University of Nevada Los Vegas, H. J. Summer III of Penn State University, Rajeev Madhavan Nair of Iowas State University, Ali P. Gordon of University of Central Florida, Robert Jackson of Auburn University, Cara Coad of Colorado School of Mines, Burford J. Furman of San Jose State University, Steven J. Covey of St. Cloud State University, Nathan Crane of University of Central Florida, César Augusto Álvarez Vargas of Universidad Autonoma de Manizales, Caldas, Colombia, Naser Nawayseh of Dhofar University, Oman, Hodge E. Jenkins of Mercer University, John Lee of San Jose State University, Mahmoud Kadkhodaei of Isfahan University of Technology, Steve Searcy of Texas A&M University, Yesh P. Singh of University of Texas at San Antonio, and Osornio C. Cuitláhuac of Universidad Iberoamericana Santa Fe, Mexico.

The author is greatly indebted to Thomas A. Cook, Professor Emeritus, Mercer University, who did the Solutions Manual for this book, updated the Mathcad examples, and contributed most of the new problem sets for this edition. Thanks also to Dr. Adriana Hera of Worcester Polytechnic Institute who updated the MATLAB and Excel models of all the examples and case studies and thoroughly vetted their correctness.

Finally, Nancy Norton, my infinitely patient wife for the past fifty years, deserves renewed kudos for her unfailing support and encouragement during many summers of "book widowhood." I could not have done it without her.

Every effort has been made to eliminate errors from this text. Any that remain are the author's responsibility. He will greatly appreciate being informed of any errors that still remain so they can be corrected in future printings. An e-mail to norton@wpi.edu will be sufficient.

Robert L. Norton
Mattapoisett, Mass.
August 1, 2009

Part

I

FUNDAMENTALS

1

INTRODUCTION TO DESIGN

Learning without thought is labor lost;
thought without learning is perilous.

CONFUCIUS, 6TH CENTURY B.C.

1.1 DESIGN

What is design? Wallpaper is designed. You may be wearing "designer" clothes. Automobiles are "designed" in terms of their external appearance. The term *design* clearly encompasses a wide range of meaning. In the above examples, design refers primarily to the object's aesthetic appearance. In the case of the automobile, all of its other aspects also involve design. Its mechanical internals (engine, brakes, suspension, etc.) must be designed, more likely by engineers than by artists, though even the engineer gets to exhibit some artistry when designing machinery.

The word design is from the Latin word ***designare*** meaning "*to designate, or mark out*." Webster's dictionary gives several definitions of the word **design**, the most applicable of which is "*to outline, plot, or plan as action or work . . . to conceive, invent, contrive*." We are more concerned here with engineering design than with artistic design. **Engineering design** can be defined as "*The process of applying the various techniques and scientific principles for the purpose of defining a device, a process, or a system in sufficient detail to permit its realization*."

Machine Design

This text is concerned with one aspect of engineering design—**machine design**. Machine design deals with the creation of machinery that works safely, reliably, and well. A **machine** can be defined in many ways. The Random House dictionary[1] lists twelve definitions, among which are these two:

Machine

1. An apparatus consisting of interrelated units, or
2. A device that modifies force or motion.

The **interrelated parts** referred to in the definition are also sometimes called **machine elements** in this context. The notion of **useful work** is basic to a machine's function, as there is almost always some energy transfer involved. The mention of **forces** and **motion** is also critical to our concerns, as, in converting energy from one form to another, machines **create motion** and **develop forces**. It is the engineer's task to define and calculate those motions, forces, and changes in energy in order to determine the sizes, shapes, and materials needed for each of the interrelated parts in the machine. This is the essence of **machine design**.

While one must, of necessity, design a machine one part at a time, it is crucial to recognize that each part's function and performance (and thus its design) are dependent on many other interrelated parts within the same machine. Thus, we are going to attempt to "design the whole machine" here, rather than simply designing individual elements in isolation from one another. To do this we must draw upon a common body of engineering knowledge encountered in previous courses, e.g., statics, dynamics, mechanics of materials (stress analysis), and material properties. Brief reviews and examples of these topics are included in the early chapters of this book.

The ultimate goal in machine design is to size and shape the parts (machine elements) and choose appropriate materials and manufacturing processes so that the resulting machine can be expected to perform its intended function without failure. This requires that the engineer be able to calculate and predict the mode and conditions of failure for each element and then design it to prevent that failure. This in turn requires that a **stress and deflection analysis** be done for each part. Since stresses are a function of the applied and inertial loads, and of the part's geometry, an analysis of the forces, moments, torques, and the dynamics of the system must be done before the stresses and deflections can be completely calculated.

If the "machine" in question has no moving parts, then the design task becomes much simpler, because only a static force analysis is required. But if the machine has no moving parts, it is not much of a machine (and doesn't meet the definition above); it is then a **structure**. Structures also need to be designed against failure, and, in fact, large external structures (bridges, buildings, etc.) are also subjected to dynamic loads from wind, earthquakes, traffic, etc., and thus must also be designed for these conditions. Structural dynamics is an interesting subject but one which we will not address in this text. We will concern ourselves with the problems associated with machines that move. If the machine's motions are very slow and the accelerations negligible, then a static force analysis will suffice. But if the machine has significant accelerations within it, then a dynamic force analysis is needed and the accelerating parts become "victims of their own mass."

In a static structure, such as a building's floor, designed to support a particular weight, the safety factor of the structure can be increased by adding appropriately distributed material to its structural parts. Though it will be heavier (more "dead" weight), if properly designed it may nevertheless carry more "live" weight (payload) than it did before, still without failure. In a dynamic machine, adding weight (mass) to moving parts may have the opposite effect, reducing the machine's safety factor, its allowable

Title-page photograph courtesy of Boeing Airplane Co. Inc., Seattle, Wash.

speed, or its payload capacity. This is because some of the loading that creates stresses in the moving parts is due to the inertial forces predicted by **Newton's second law**, $F = ma$. Since the accelerations of the moving parts in the machine are dictated by its kinematic design and by its running speed, adding mass to moving parts will increase the inertial loads on those same parts unless their kinematic accelerations are reduced by slowing its operation. Even though the added mass may increase the strength of the part, that benefit may be reduced or cancelled by the resultant increases in inertial forces.

Iteration

Thus, we face a dilemma at the initial stages of machine design. Generally, before reaching the stage of sizing the parts, the kinematic motions of the machine will have already been defined. External forces provided by the "outside world" on the machine are also often known. Note that in some cases, the external loads on the machine will be very difficult to predict—for example, the loads on a moving automobile. The designer cannot predict with accuracy what environmental loads the user will subject the machine to (potholes, hard cornering, etc.) In such cases, statistical analysis of empirical data gathered from actual testing can provide some information for design purposes.

What remain to be defined are the inertial forces that will be generated by the known kinematic accelerations acting on the as yet undefined masses of the moving parts. The dilemma can be resolved only by **iteration**, which means *to repeat, or to return to a previous state*. We must assume some trial configuration for each part, use the mass properties (mass, *CG* location, and mass moment of inertia) of that trial configuration in a dynamic force analysis to determine the forces, moments, and torques acting on the part, and then use the cross-sectional geometry of the trial design to calculate the resulting stresses. In general, accurately determining all the loads on a machine is the most difficult task in the design process. If the loads are known, the stresses can be calculated.

Most likely, on the first trial, we will find that our design fails because the materials cannot stand the levels of stress presented. We must then redesign the parts (iterate) by changing shapes, sizes, materials, manufacturing processes, or other factors in order to reach an acceptable design. It is generally not possible to achieve a successful result without making several iterations through this design process. Note also that a change to the mass of one part will also affect the forces applied to parts connected to it and thus require their redesign also. It is truly the design of **interrelated parts**.

1.2 A DESIGN PROCESS*

The process of design is essentially an exercise in applied creativity. Various "design processes" have been defined to help organize the attack upon the "unstructured problem," i.e., one for which the problem definition is vague and for which many possible solutions exist. Some of these design process definitions contain only a few steps and others a detailed list of 25 steps. One version of a design process is shown in Table 1-1, which lists ten steps. The initial step, **Identification of Need**, usually consists of an ill-defined and vague problem statement. The development of **Background Research** information (step 2) is necessary to fully define and understand the problem, after which it is possible to restate the **Goal** (step 3) in a more reasonable and realistic way than in the original problem statement.

* Adapted from Norton, *Design of Machinery*, 3ed. McGraw-Hill, New York, 2004, with the publisher's permission.

Table 1-1 A Design Process

1	Identification of need
2	Background research
3	Goal statement
4	Task specifications
5	Synthesis
6	Analysis
7	Selection
8	Detailed design
9	Prototyping and testing
10	Production

Step 4 calls for the creation of a detailed set of **Task Specifications** which bound the problem and limit its scope. The **Synthesis** step (5) is one in which as many alternative design approaches as possible are sought, usually without regard (at this stage) for their value or quality. This is also sometimes called the **Ideation and Invention** step in which the largest possible number of creative solutions are generated.

In step 6, the possible solutions from the previous step are **Analyzed** and either accepted, rejected, or modified. The most promising solution is **Selected** at step 7. Once an acceptable design is selected, the **Detailed Design** (step 8) can be done, in which all the loose ends are tied up, complete engineering drawings made, vendors identified, manufacturing specifications defined, etc. The actual construction of the working design is first done as a **Prototype** in step 9 and finally in quantity in **Production** at step 10. A more complete discussion of this design process can be found in reference 2, and a number of references on the topics of creativity and design are provided in the bibliography at the end of this chapter.

The above description may give an erroneous impression that this process can be accomplished in a linear fashion as listed. On the contrary, **iteration is required within the entire process**, moving from any step back to any previous step, in all possible combinations, and doing this repeatedly. The best ideas generated at step 5 will invariably be discovered to be flawed when later analyzed. Thus a return to at least the Ideation step will be necessary in order to generate more solutions. Perhaps a return to the Background Research phase may be necessary to gather more information. The Task Specifications may need to be revised if it turns out that they were unrealistic. In other words, anything is “fair game” in the design process, including a redefinition of the problem, if necessary. One cannot design in a linear fashion. It’s three steps forward and two (or more) back, until you finally emerge with a working solution.

Theoretically, we could continue this iteration on a given design problem forever, continually creating small improvements. Inevitably, the incremental gains in function or reductions in cost will tend toward zero with time. At some point, we must declare the design “good enough” and ship it. Often someone else (most likely, the boss) will snatch it from our grasp and ship it over our protests that it isn’t yet “perfect.” Machines that have been around a long time and that have been improved by many designers reach a level of “perfection” that makes them difficult to improve upon. One example is the ordinary bicycle. Though inventors continue to attempt to improve this machine, the basic design has become fairly static after more than a century of development.

In machine design, the early design-process steps usually involve the **Type Synthesis** of suitable kinematic configurations which can provide the necessary motions. Type synthesis involves the choice of the *type of mechanism best suited to the problem.* This is a difficult task for the student, as it requires some experience and knowledge of the various types of mechanisms that exist and that might be feasible from a performance and manufacturing standpoint. As an example, assume that the task is to design a device to track the constant-speed, straight-line motion of a part on a conveyor belt and attach a second part to it as it passes by. This has to be done with good accuracy and repeatability and must be reliable and inexpensive. You might not be aware that this task could be accomplished by any of the following devices:

- a straight-line linkage
- a cam and follower
- an air cylinder
- a hydraulic cylinder
- a robot
- a solenoid

Each of these solutions, while possible, may not be optimal or even practical. Each has good and bad points. The straight-line linkage is large and may have undesirable accelerations, the cam and follower is expensive but is accurate and repeatable. The air cylinder is inexpensive but noisy and unreliable. The hydraulic cylinder and the robot are more expensive. The inexpensive solenoid has high impact loads and velocities. So, the choice of device type can have a big effect on design quality. A bad choice at the type-synthesis stage can create major problems later on. The design might have to be changed after completion, at great expense. Design is essentially an exercise in trade-offs. There is usually no clear-cut solution to a real engineering design problem.

Once the type of required mechanism is defined, its detailed kinematics must be synthesized and analyzed. The motions of all moving parts and their time derivatives through acceleration must be calculated in order to be able to determine the dynamic forces on the system. (See reference 2 for more information on this aspect of machine design.)

In the context of machine design addressed in this text, we will not exercise the entire design process as described in Table 1-1. Rather, we will propose examples, problems, and case studies that already have had steps 1–4 defined. The type synthesis and kinematic analysis will already be done, or at least set up, and the problems will be structured to that degree. The tasks remaining will largely involve steps 5 through 8, with a concentration on **synthesis** (step 5) and **analysis** (step 6).

Synthesis and analysis are the "two faces" of machine design, like two sides of the same coin. **Synthesis** means *to put together* and **analysis** means *to decompose, to take apart, to resolve into its constituent parts.* Thus they are opposites, but they are symbiotic. We cannot take apart "nothing," thus we must first synthesize something in order to analyze it. When we analyze it, we will probably find it lacking, requiring further synthesis, and then further analysis *ad nauseam*, finally iterating to a better solution. You will need to draw heavily upon your understanding of statics, dynamics, and mechanics of materials to accomplish this.

1.3 PROBLEM FORMULATION AND CALCULATION

It is extremely important for every engineer to develop good and careful computational habits. Solving complicated problems requires an organized approach. Design problems also require good record-keeping and documentation habits in order to record the many assumptions and design decisions made along the way so that the designer's thought process can be later reconstructed if redesign is necessary.

A suggested procedure for the designer is shown in Table 1-2, which lists a set of subtasks appropriate to most machine-design problems of this type. These steps should be documented for each problem in a neat fashion, preferably in a bound notebook in order to maintain their chronological order.*

Definition Stage

In your design notebook, first **Define the Problem** clearly in a concise statement. The "**givens**" for the particular task should be clearly listed, followed by a record of the **assumptions** made by the designer about the problem. Assumptions expand upon the given (known) information to further constrain the problem. For example, one might assume the effects of friction to be negligible in a particular case, or assume that the weight of the part can be ignored because it will be small compared to the applied or dynamic loads expected.

Preliminary Design Stage

Once the general constraints are defined, some **Preliminary Design Decisions** must be made in order to proceed. The reasons and justifications for these decisions should be documented. For example, we might decide to try a solid, rectangular cross section for a connecting link and choose aluminum as a trial material. On the other hand, if we recognized from our understanding of the problem that this link would be subjected to significant accelerations of a time-varying nature that would repeat for millions of cycles, a better design decision might be to use a hollow or I-beam section in order to reduce its mass and also to choose steel for its infinite fatigue life. Thus, these design decisions can have significant effect on the results and will often have to be changed or abandoned as we iterate through the design process. It has often been noted that 90% of a design's characteristics may be determined in the first 10% of the total project time, during which these preliminary design decisions are made. If they are bad decisions, it may not be possible to save the bad design through later modifications without essentially starting over. The preliminary design concept should be documented at this stage with clearly drawn and labeled **Design Sketches** that will be understandable to another engineer or even to oneself after some time has passed.

* If there is a possibility of a patentable invention resulting from the design, then the notebook should be permanently bound (not loose-leaf), and its pages should be consecutively numbered, dated, and witnessed by someone who understands the technical content.

Table 1-2 Problem Formulation and Calculation

1	Define the problem	Definition stage
2	State the givens	
3	Make appropriate assumptions	
4	Preliminary design decisions	Preliminary design stage
5	Design sketches	
6	Mathematical models	Detailed design stage
7	Analysis of the design	
8	Evaluation	
9	Document results	Documentation stage

Detailed Design Stage

With a tentative design direction established we can create one or more **engineering (**mathematical) **models** of the element or system in order to analyze it. These models will usually include a loading model consisting of free-body diagrams which show all forces, moments, and torques on the element or system and the appropriate equations for their calculation. Models of the stress and deflection states expected at locations of anticipated failure are then defined with appropriate stress and deflection equations.

Analysis of the design is then done using these models and the safety or failure of the design determined. The results are **evaluated** in conjunction with the properties of the chosen **engineering materials** and a decision made whether to proceed with this design or iterate to a better solution by returning to an earlier step of the process.

Documentation Stage

Once sufficient iteration through this process provides satisfactory results, the **documentation** of the element's or system's design should be completed in the form of detailed engineering drawings, material and manufacturing specifications, etc. If properly approached, a great deal of the documentation task can be accomplished concurrent with the earlier stages simply by keeping accurate and neat records of all assumptions, computations, and design decisions made throughout the process.

1.4 THE ENGINEERING MODEL

The success of any design is highly dependent on the validity and appropriateness of the engineering models used to predict and analyze its behavior in advance of building any hardware. Creating a useful engineering model of a design is probably the most difficult and challenging part of the whole process. Its success depends a great deal on experience as well as skill. Most important is a thorough understanding of the first principles and fundamentals of engineering. The engineering model that we are describing here is an amorphous thing which may consist of some sketches of the geometric configuration and some equations that describe its behavior. It is a mathematical model that describes the physical behavior of the system. This engineering model invariably requires the use of computers to exercise it. Using computer tools for analyzing engineering models is discussed in the next section. A physical model or prototype usually comes later in the process and is necessary to prove the validity of the engineering model through experiments.

Estimation and First-Order Analysis

The value of making even very simplistic engineering models of your preliminary designs cannot be overemphasized. Often, at the outset of a design, the problem is so loosely and poorly defined that it is difficult to develop a comprehensive and thorough model in the form of equations that fully describe the system. The engineering student is used to problems that are fully structured, of a form such as "*Given A, B, and C, find D*." If one can identify the appropriate equations (model) to apply to such a problem, it is relatively easy to determine an answer (which might even match the one in the back of the book).

Real-life engineering design problems are not of this type. They are very **unstructured** and must be structured by you before they can be solved. Also, there is no "*back of the book*" to refer to for the answer.* This situation makes most students and beginning engineers very nervous. They face the "blank paper syndrome," not knowing where to begin. A useful strategy is to recognize that

1 You must begin somewhere.

2 Wherever you begin, it will probably not be the "best" place to do so.

3 The magic of iteration will allow you to back up, improve your design, and eventually succeed.

With this strategy in mind, you can feel free to make some estimation of a design configuration at the outset, assume whatever limiting conditions you think appropriate, and do a "first-order analysis," one that will be only an estimate of the system's behavior. These results will allow you to identify ways to improve the design. Remember that it is preferable to get a reasonably approximate but quick answer that tells you whether the design does or doesn't work rather than to spend more time getting the same result to more decimal places. With each succeeding iteration, you will improve your understanding of the problem, the accuracy of your assumptions, the complexity of your model, and the quality of your design decisions. Eventually, you will be able to refine your model to include all relevant factors (or identify them as irrelevant) and obtain a higher-order, final analysis in which you have more confidence.

The Engineering Sketch

A sketch of the concept is often the starting point for a design. This may be a freehand sketch, but it should always be made reasonably to scale in order to show realistic geometric proportions. This sketch often serves the primary purpose of communicating the concept to other engineers and even to yourself. It is one thing to have a vague concept in mind and quite another to define it in a sketch. This sketch should, at a minimum, contain three or more orthographic views, aligned according to proper drafting convention, and may also include an isometric or trimetric view. Figure 1-1 shows a freehand sketch of a simple design for one subassembly of a trailer hitch for a tractor.

* A student once commented that "*Life is an odd-numbered problem*." This (slow) author had to ask for an explanation, which was: "*The answer is not in the back of the book.*"

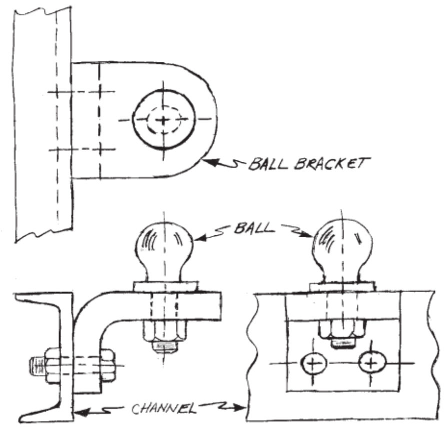


FIGURE 1-1

A Freehand Sketch of a Trailer Hitch Assembly for a Tractor

While often incomplete in terms of detail needed for manufacture, the engineering sketch should contain enough information to allow the development of an engineering model for design and analysis. This may include critical, if approximate, dimensional information, some material assumptions, and any other data germane to its function that is needed for further analysis. The engineering sketch captures some of the givens and assumptions made, even implicitly, at the outset of the design process.

1.5 COMPUTER-AIDED DESIGN AND ENGINEERING

The computer has created a true revolution in engineering design and analysis. Problems whose solution methods have literally been known for centuries but that only a generation ago were practically unsolvable due to their high computational demands can now be solved in minutes on inexpensive microcomputers. Tedious graphical solution methods were developed in the past to circumvent the lack of computational power available from slide rules. Some of these graphical solution methods still have value in that they can show the results in an understandable form. But one can no longer "do engineering" without using its latest and most powerful tool, the computer.

Computer-Aided Design (CAD)

As the design progresses, the crude freehand sketches made at the earliest stages will be supplanted by formal drawings made either with conventional drafting equipment or, as is increasingly common, with computer-aided design or computer-aided drafting software. If the distinction between these two terms (both of which share the acronym CAD) was ever clear (a subject for debate which will be avoided here), then that distinction is fading as more sophisticated CAD software becomes available. The original CAD systems of a generation ago were essentially drafting tools that allowed the creation of computer-generated multiview drawings similar to those done for centuries before by hand on a drafting board. The data stored in these early CAD systems were strictly two-dimensional representations of the orthographic projections of the part's true 3-D geometry. Only the edges of the part were defined in the database. This is called a **wireframe model**. Some 3-D CAD packages use wireframe representation as well.

Present versions of most CAD software packages allow (and sometimes require) that the geometry of the parts be encoded in a 3-D data base as **solid models**. In a solid model the edges and the faces of the part are defined. From this 3-D information, the conventional 2-D orthographic views can be automatically generated if desired. The major advantage of creating a 3-D solid-model geometric data base for any design is that its mass-property information can be rapidly calculated. (This is not possible in a 2-D or 3-D wireframe model.) For example, in designing a machine part, we need to determine the location of its center of gravity (CG), its mass, its mass moment of inertia, and its cross-sectional geometries at various locations. Determining this information from a 2-D model must be done outside the CAD package. That is tedious to do and can only be approximate when the geometry is complex. But, if the part is designed in a solid modeling CAD system such as *ProEngineer,*[7] *Unigraphics,*[4] or one of many others, the mass properties can be calculated for the most complicated part geometries.

Solid modeling systems usually provide an interface to one or more Finite Element Analysis (FEA) programs and allow direct transfer of the model's geometry to the FEA package for stress, vibration, and heat transfer analysis. Some CAD systems include a mesh-generation feature which creates the FEA mesh automatically before sending the data to the FEA software. This combination of tools provides an extremely powerful means to obtain superior designs whose stresses are more accurately known than would be possible by conventional analysis techniques when the geometry is complex.

While it is highly likely that the students reading this textbook will be using CAD tools including finite element or boundary element analysis (BEA) methods in their professional practice, it is still necessary that the fundamentals of applied stress analysis be thoroughly understood. That is the purpose of this text. FEA techniques will be discussed in Chapters 4 and 8 but will not be emphasized in this text. Rather we will concentrate on the classical stress-analysis techniques in order to lay the foundation for a thorough understanding of the fundamentals and their application to machine design.

FEA and BEA methods are rapidly becoming the methods of choice for the solution of complicated stress-analysis problems. However, there is danger in using those techniques without a solid understanding of the theory behind them. These methods will always give *some* results. Unfortunately, those results can be incorrect if the problem was not well formulated and well meshed with proper boundary conditions applied. Being able to recognize incorrect results from a computer-aided solution is extremely important to the success of any design. Chapter 8 provides a brief introduction to FEA. The student should take courses in FEA and BEA to become familiar with these tools.

Figure 1-2 shows a solid model of the ball bracket from Figure 1-1 that was created in a CAD software package. The shaded, isometric view in the upper right corner shows that the solid volume of the part is defined. The other three views show orthographic projections of the part. Figure 1-3 shows the mass-properties data which are calculated by the software. Figure 1-4 shows a wireframe rendering of the same part generated from the solid geometry data base. A wireframe version is used principally to speed up the screen-drawing time when working on the model. There is much less wireframe display information to calculate than for the solid rendering of Figure 1-2.

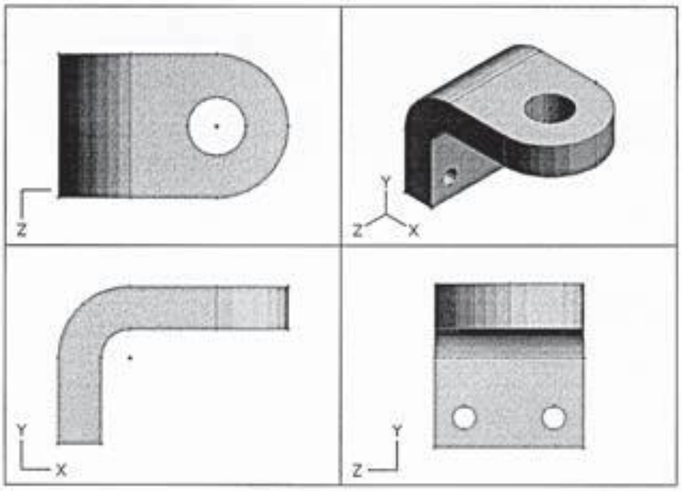


FIGURE 1-2

A CAD Solid Model of the Ball Bracket from the Trailer Hitch Assembly of Figure 1-1

Figure 1-5 shows a fully dimensioned, orthographic, multiview drawing of the ball bracket that was generated in the CAD software package. Another major advantage of creating a solid model of a part is that the dimensional and tool-path information needed for its manufacture can be generated in the CAD system and sent over a network to a computer-controlled machine on the manufacturing floor. This feature allows the production of parts without the need for paper drawings such as Figure 1-5. Figure 1-6 shows the same part after a finite element mesh was applied to it by the CAD software before sending it to the FEA software for stress analysis.

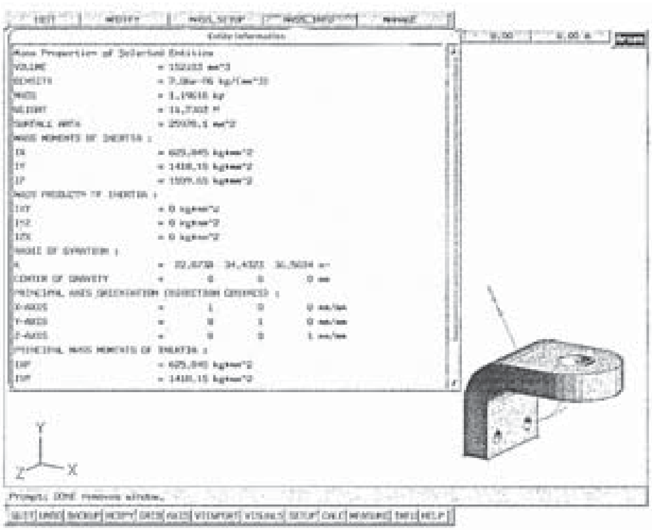


FIGURE 1-3

Mass Properties of the Ball Bracket Calculated Within the CAD System from Its Solid Model

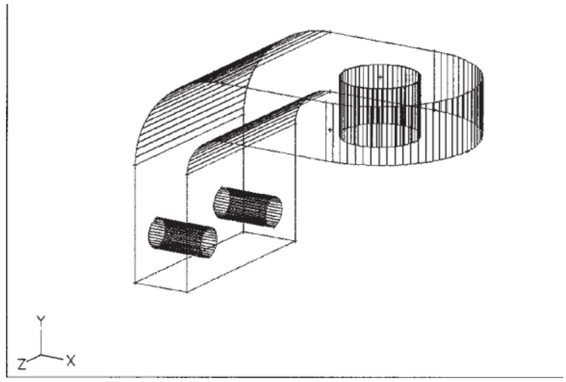


FIGURE 1-4

A Wireframe Representation of the Ball Bracket Generated from Its Solid Model in a CAD System

Computer-Aided Engineering (CAE)

The techniques generally referred to above as CAD are a subset of the more general topic of computer-aided engineering (CAE), which term implies that more than just the geometry of the parts is being dealt with. However, the distinctions between CAD and CAE continue to blur as more sophisticated software packages become available. In fact, the description of the use of a solid modeling CAD system and an FEA package together as described in the previous section is an example of CAE. When some analysis of forces, stresses, deflections, or other aspects of the physical behavior of the design

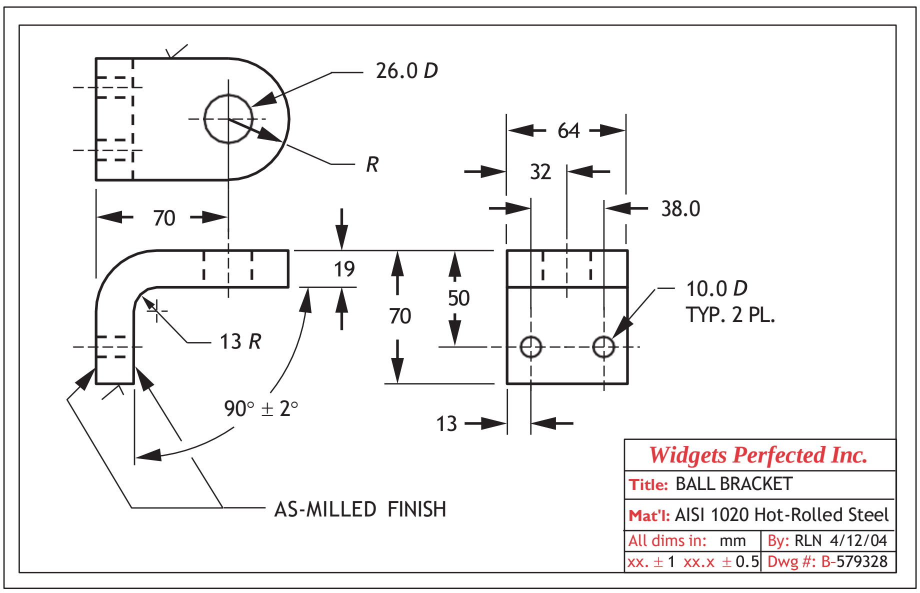


FIGURE 1-5

A Dimensioned, 3-View Orthographic Drawing Done in a 2-D CAD Drawing Package

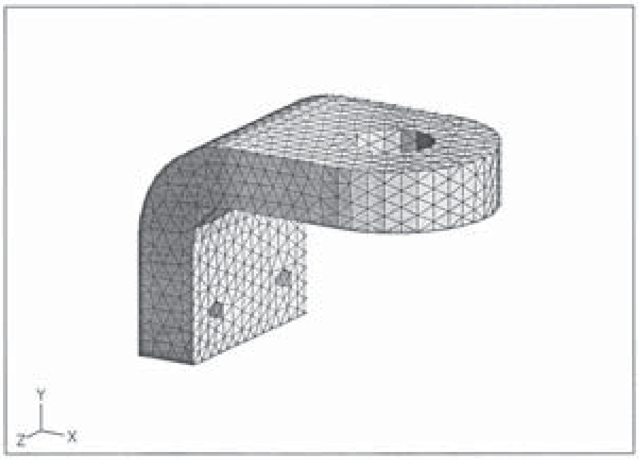


FIGURE 1-6

An FEA Mesh Applied to the Solid Model of the Ball Bracket in the CAD System

is included, with or without the solid geometry aspects, the process is called CAE. Many commercial software packages do one or more aspects of CAE. The FEA and BEA software packages mentioned above are in this category. See Chapter 8 for more information on FEA. Dynamic force simulations of mechanisms can be done with such packages as *ADAMS*[5] and *Working Model*.[6] Some software packages such as *ProEngineer*,[7] *Solidworks*,[12] *Unigraphics*,[4] and others combine aspects of a CAD system with general analysis capabilities. These constraint-based programs allow constraints to be applied to the design which can control the part geometry as the design parameters are varied.

Other classes of tools for CAE are equation solvers such as *MATLAB*[11], *Mathcad*,[9] *TK Solver*,[8] and spreadsheets such as *Excel*.[10] These are general-purpose tools that will allow any combination of equations to be encoded in a convenient form and then will manipulate the equation set (i.e., the engineering model) for different trial data and conveniently display tabular and graphic output. Equation solvers are invaluable for the solution of force, stress, and deflection equations in machine-design problems because they allow rapid "what-if" calculations to be done. The effects of dimensional or material changes on the stresses and deflections in the part can be seen instantly. In the absence of a true solid modeling system, an equation solver also can be used to approximate the part's mass properties while iterating the geometry and material properties of trial part designs. Rapid iteration to an acceptable solution is thus enhanced.

The CD-ROM included with the text contains a large number of models for various equation solvers that support the examples and case studies presented in the text. Introductions to the use of *TK Solver* and *Mathcad* along with examples of their use are provided as PDF files on the CD-ROM. In addition, some custom-written computer programs, MOHR, CONTACT, ASDEQ, FOURBAR, FIVEBAR, SIXBAR, SLIDER, DYNACAM, and MATRIX are provided on the CD-ROM to aid in the calculation of dynamic loads and stresses when solving the open-ended design problems assigned.

However, one must be aware that these computer tools are just tools and are not a substitute for the human brain. Without a thorough understanding of the engineering fundamentals on the part of the user, the computer will not give good results. Garbage in, garbage out. *Caveat Lector*.

Computational Accuracy

Computers and calculators make it very easy to obtain numerical answers having many significant figures. Before writing down all those digits, you are advised to recall the accuracy of your initial assumptions and given data. If, for example, your applied loads were known to only two significant figures, it is incorrect and misleading to express the calculated stresses to more significant figures than your input data possessed. However, it is valid and appropriate to make all intermediate calculations to the greatest accuracy available in your computational tools. This will minimize computational round-off errors. But, when done, round off the results to a level consistent with your known or assumed data.

1.6 THE ENGINEERING REPORT*

Communication of your ideas and results is a very important aspect of engineering. Many engineering students picture themselves in professional practice spending most of their time doing calculations of a nature similar to those they have done as students. Fortunately, this is seldom the case, as it would be very boring. Actually, engineers spend a large percentage of their time communicating with others, either orally or in writing. Engineers write proposals and technical reports, give presentations, and interact with support personnel. When your design is done, it is usually necessary to present the results to your client, peers, or employer. The usual form of presentation is a formal engineering report. In addition to a written description of the design, these reports will usually contain engineering drawings or sketches as described earlier, as well as tables and graphs of data calculated from the engineering model.

It is very important for engineering students to develop their communication skills. *You may be the cleverest person in the world, but no one will know that if you cannot communicate your ideas clearly and concisely*. In fact, if you cannot explain what you have done, you probably don't understand it yourself. The design-project assignments in Chapter 9 are intended to be written up in formal engineering reports to give you some experience in this important skill of technical communication. Information on writing engineering reports can be found in the suggested readings listed in the bibliography.

1.7 FACTORS OF SAFETY AND DESIGN CODES

The quality of a design can be measured by many criteria. It is always necessary to calculate one or more factors of safety to estimate the likelihood of failure. There may be legislated, or generally accepted, design codes that must be adhered to as well.

* Excerpted from Norton, *Design of Machinery*, 3ed McGraw-Hill, New York, 2004, with the publisher's permission.

Factor of Safety*

A factor of safety or safety factor can be expressed in many ways. It is typically a ratio of two quantities that have the same units, such as strength/stress, critical load/applied load, load to fail part/expected service overload, maximum cycles/applied cycles, or maximum safe speed/operating speed. A safety factor is always unitless.

The form of expression for a safety factor can usually be chosen based on the character of loading on the part. For example, consider the loading on the side wall of a cylindrical water tower that can never be "more than full" of a liquid of known density within a known temperature range. Since this loading is highly predictable over time, a comparison of the strength of the material to the stress in the wall of a full tank might be appropriate as a safety factor. Note in this example that the possibility of rust reducing the thickness of the wall over time must be considered. (See Section 4.17 for a discussion of stresses in cylinder walls and Section 7.6 for a discussion of corrosion.)

If this cylindrical water tower is standing on legs loaded as columns, then a safety factor for the legs based on a ratio of the column's critical buckling load over the applied load from a full water tower would be appropriate. (See Section 4.16 for a discussion of column buckling.)

If a part is subjected to loading that varies cyclically with time, it may experience fatigue failure. The resistance of a material to some types of fatigue loading can be expressed as a maximum number of cycles of stress reversal at a given stress level. In such cases, it may be appropriate to express the safety factor as a ratio of the maximum number of cycles to expected material failure over the number of cycles applied to the part in service for its desired life. (See Chapter 6 for a discussion of fatigue-failure phenomena and several approaches to the calculation of safety factors in such situations.)

The safety factor of a part such as a rotating sheave (pulley) or flywheel is often expressed as a ratio of its maximum safe speed over the highest expected speed in service. In general, if the stresses in the parts are a linear function of the applied service loads and those loads are predictable, then a safety factor expressed as strength/stress or failure load/applied load will give the same result. Not all situations fit these criteria. Some require a nonlinear ratio. A column is one example, because its stresses are a nonlinear function of the loading (see Section 4.16). Thus a critical (failure) load for the particular column must be calculated for comparison to the applied load.

Another complicating factor is introduced when the magnitudes of the expected applied loads are not accurately predictable. This can be true in virtually any application in which the use (and thus the loading) of the part or device is controlled by humans. For example, there is really no way to prevent someone from attempting to lift a 10-ton truck with a jack designed to lift a 2-ton automobile. When the jack fails, the manufacturer (and designer) may be blamed even though the failure was probably due more to the "nut behind the jack handle." In situations where the user may subject the device to overloading conditions, an assumed overload may have to be used to calculate a safety factor based on a ratio of the load that causes failure over the assumed service overload. Labels warning against inappropriate use may be needed in these situations as well.

Since there may be more than one potential mode of failure for any machine element, it can have more than one value of safety factor N. The smallest value of N for

* Also called *safety factor*. We will use both terms interchangeably in this text.

any part is of greatest concern, since it predicts the most likely mode of failure. When N becomes reduced to 1, the stress in the part is equal to the strength of the material (or the applied load is equal to the load that fails it, etc.) and failure occurs. Therefore, we desire N to be always greater than 1.

Choosing a Safety Factor

Choosing a safety factor is often a confusing proposition for the beginning designer. The safety factor can be thought of as a measure of the designer's uncertainty in the analytical models, failure theories, and material-property data used, and should be chosen accordingly. How much greater than one N must be depends on many factors, including our level of confidence in the model on which the calculations are based, our knowledge of the range of possible in-service loading conditions, and our confidence in the available material-strength information. If we have done extensive testing on physical prototypes of our design to prove the validity of our engineering model and of the design, and have generated test data on the particular material's strengths, then we can afford to use a smaller safety factor. If our model is less well proven or the material-property information is less reliable, a larger N is in order. In the absence of any design codes that may specify N for particular cases, the choice of factor of safety involves engineering judgment. A reasonable approach is to determine the largest loads expected in service (including possible overloads) and the minimum expected material strengths and base the safety factors on these data. The safety factor then becomes a reasonable measure of uncertainty.

If you fly, it may not give you great comfort to know that safety factors for commercial aircraft are in the range of 1.2 to 1.5. Military aircraft can have $N < 1.1$, but their crews wear parachutes. (Test pilots deserve their high salaries.) Missiles have $N = 1$ but have no crew and aren't expected to return anyway. These small factors of safety in aircraft are necessary to keep weight low and are justified by sophisticated analytical modeling (usually involving FEA), testing of the actual materials used, extensive testing of prototype designs, and rigorous in-service inspections for incipient failures of the equipment. The opening photograph of this chapter shows an elaborate test rig used by the Boeing Aircraft Co. to mechanically test the airframe of full-scale prototype or production aircraft by applying dynamic forces and measuring their effects.

It can be difficult to predict the kinds of loads that an assembly will experience in service, especially if those loads are under the control of the end-user, or Mother Nature. For example, what loads will the wheel and frame of a bicycle experience? It depends greatly on the age, weight, and recklessness of the rider, whether used on- or off-road, etc. The same problem of load uncertainty exists with all transportation equipment, ships, aircraft, automobiles, etc. Manufacturers of these devices engage in extensive test programs to measure typical service loads. See Figures 3-16 (p. 106) and 6-7 (p. 315) for examples of such service-load data.

Some guidelines for the choice of a safety factor in machine design can be defined based on the quality and appropriateness of the material-property data available, the expected environmental conditions compared to those under which the material test data were obtained, and the accuracy of the loading- and stress-analysis models developed for the analyses. Table 1-3 shows a set of factors for ductile materials which can be chosen in each of the three categories listed based on the designer's knowledge or judg-

Table 1-3 Factors Used to Determine a Safety Factor for Ductile Materials

Information	Quality of Information	Factor
		F1
	The actual material used was tested	1.3
Material-property data	Representative material test data are available	2
available from tests	Fairly representative material test data are available	3
	Poorly representative material test data are available	5+
		F2
	Are identical to material test conditions	1.3
Environmental conditions	Essentially room-ambient environment	2
in which it will be used	Moderately challenging environment	3
	Extremely challenging environment	5+
		F3
	Models have been tested against experiments	1.3
Analytical models for	Models accurately represent system	2
loading and stress	Models approximately represent system	3
	Models are crude approximations	5+

ment of the quality of information used. The overall safety factor is then taken as the largest of the three factors chosen. Given the uncertainties involved, a safety factor typically should not be taken to more than 1 decimal place accuracy.

$$N_{ductile} \cong MAX(F1, F2, F3) \tag{1.1a}$$

The ductility or brittleness of the material is also a concern. Brittle materials are designed against the ultimate strength, so failure means fracture. Ductile materials under static loads are designed against the yield strength and are expected to give some visible warning of failure before fracture, unless cracks indicate the possibility of a fracture-mechanics failure (see Sections 5-3 and 6-5). For these reasons, the safety factor for brittle materials is often made twice that which would be used for a ductile material in the same situation:

$$N_{brittle} \cong 2 * MAX(F1, F2, F3) \tag{1.1b}$$

This method of determining a safety factor is only a guideline to obtain a starting point and is obviously subject to the judgment of the designer in selecting factors in each category. The designer has the ultimate responsibility to ensure that the design is safe. A larger safety factor than any shown in Table 1-3 may be appropriate in some circumstances.

Design and Safety Codes

Many engineering societies and government agencies have developed codes for specific areas of engineering design. Most are only recommendations, but some have the force of law. The ASME provides recommended guidelines for safety factors to be used in particular applications such as steam boilers and pressure vessels. Building codes are legislated in most U.S.A. states and cities and usually deal with publicly accessible structures or their components, such as elevators and escalators. Safety factors are sometimes specified in these codes and may be quite high. (The code for escalators in one state called for a factor of safety of 14.) Clearly, where human safety is involved, high values of N are justified. However, they come with a weight and cost penalty, as parts must often be made heavier to achieve large values of N. The design engineer must always be aware of these codes and standards and adhere to them where applicable.

The following is a partial list of engineering societies and governmental, industrial, and international organizations that publish standards and codes of potential interest to the mechanical engineer. Addresses and data on their publications can be obtained in any technical library or from the internet.

American Gear Manufacturers Association (AGMA) http://www.agma.org/

American Institute of Steel Construction (AISC) http://www.aisc.org/

American Iron and Steel Institute (AISI) http://www.steel.org/

American National Standards Institute (ANSI) http://www.ansi.org/

American Society for Metals (ASM International) http://www.asm-intl.org/

American Society of Mechanical Engineers (ASME) http://www.asme.org/

American Society of Testing and Materials (ASTM) http://www.astm.org/

American Welding Society (AWS) http://www.aws.org/

Anti-Friction Bearing Manufacturers Association (AFBMA)

International Standards Organization (ISO) http://www.iso.ch/iso/en

National Institute for Standards and Technology (NIST)* http://www.nist.gov/

Society of Automotive Engineers (SAE) http://www.sae.org/

Society of Plastics Engineers (SPE) http://www.4spe.org/

Underwriters Laboratories (UL) http://www.ul.com/

1.8 STATISTICAL CONSIDERATIONS

Nothing is absolute in engineering any more than in any other endeavor. The strengths of materials will vary from sample to sample. The actual size of different examples of the "same" part made in quantity will vary due to manufacturing tolerances. As a result, we should take the statistical distributions of these properties into account in our calculations. The published data on the strengths of materials may be stated either as minimum values or as average values of tests made on many specimens. If it is an average value, there is a 50% chance that a randomly chosen sample of that material will be weaker or stronger than the published average value. To guard against failure, we can reduce the material-strength value that we will use in our calculations to a level that will include a larger percentage of the population. To do this requires some understanding of statistical phenomena and their calculation. All engineers should have this understanding and should include a statistics course in their curriculum. We briefly discuss some of the fundamental aspects of statistics in Chapter 2.

* Formerly the *National Bureau of Standards* (*NBS*).

1.9 UNITS*

Several different systems of units are used in engineering. The most common in the United States are the **U.S. *foot-pound-second system*** (***fps***), the **U.S. *inch-pound-second system*** (***ips***), and the ***Systeme Internationale*** (***SI***). The metric ***centimeter, gram, second*** (***cgs***) system is being used more frequently in the U.S., particularly in international companies, e.g., the automotive industry. All systems are created from the choice of three of the quantities in the general expression of Newton's second law

$$F = \frac{mL}{t^2} \tag{1.2a}$$

where F is force, m is mass, L is length, and t is time. The units for any three of these variables can be chosen and the other is then derived in terms of the chosen units. The three chosen units are called *base units*, and the remaining one is a *derived unit*.

Most of the confusion that surrounds the conversion of computations between either one of the U.S. systems and the *SI* system is due to the fact that the *SI* system uses a different set of base units than the U.S. systems. Both U.S. systems choose ***force***, *length*, and *time* as the base units. Mass is then a derived unit in the U.S. systems, which are referred to as *gravitational systems* because the value of mass is dependent on the local gravitational constant. The *SI* system chooses ***mass***, *length*, and *time* as the base units, and force is the derived unit. *SI* is then referred to as an *absolute system*, since the mass is a base unit whose value is not dependent on local gravity.

The **U.S. *foot-pound-second*** (***fps***) system requires that all lengths be measured in feet (ft), forces in pounds (lb), and time in seconds (sec). Mass is then derived from Newton's law as

$$m = \frac{Ft^2}{L} \tag{1.2b}$$

and its units are pounds seconds squared per **foot** (lb sec^2/ft) = **slugs**.

The **U.S. *inch-pound-second*** (***ips***) system requires that all lengths be measured in inches (in), forces in pounds (lb), and time in seconds (sec). Mass is still derived from Newton's law, equation 1.2*b*, but the units are now

pounds seconds squared per **inch** (lb sec^2/in) = **blobs**†

This mass unit is not slugs! It is worth twelve slugs or one "blob"!

Weight is defined as the force exerted on an object by gravity. Probably the most common units error that students make is to mix up these two unit systems (*fps* and *ips*) when converting weight units (which are pounds force) to mass units. Note that the gravitational acceleration constant (g or g_c) on earth at sea level is approximately 32.17 **feet** per second squared, which is equivalent to 386 **inches** per second squared. The relationship between mass and weight is

mass = weight / gravitational acceleration

$$m = \frac{W}{g_c} \tag{1.3}$$

* Excerpted from Norton, *Design of Machinery*, 3ed, 2004, McGraw-Hill, New York, with the publisher's permission.

† It is unfortunate that the mass unit in the *ips* system has never officially been given a name such as the term *slug* used for mass in the *fps* system. The author boldly suggests (with tongue only slightly in cheek) that this unit of mass in the *ips* system be called a *blob* (bl) to distinguish it more clearly from the *slug* (sl), and to help the student avoid some of the common errors listed below. Twelve *slugs* = one *blob*. *Blob* does not sound any sillier than slug, is easy to remember, implies mass, and has a convenient abbreviation (bl) which is an anagram for the abbreviation for pound (lb). Besides, if you have ever seen a garden slug, you know it looks just like a "*little blob*."

1

It should be obvious that if you measure all your lengths in **inches** and then use $g = g_c = 32.17$ **feet**/sec^2 to compute mass, you will have an error of a *factor of 12* in your results. This is a serious error, large enough to crash the airplane you designed. Even worse off is the student who neglects to convert weight to mass *at all*. The results of this calculation will have an error of either 32 or 386, which is enough to sink the ship!*

The value of mass is needed in Newton's second-law equation to determine forces due to accelerations:

$$F = ma \tag{1.4a}$$

The units of mass in this equation are either *g, kg, slugs*, or *blobs* depending on the units system used. Thus, in either English system, the weight W (lb_f) must be divided by the acceleration due to gravity g_c as indicated in equation 1.3 to get the proper mass quantity for equation 1.4*a*.

Adding further to the confusion is the common use of the unit of ***pounds mass*** (lb_m). This unit, often used in fluid dynamics and thermodynamics, comes about through the use of a slightly different form of Newton's equation:

$$F = \frac{ma}{g_c} \tag{1.4b}$$

where m = mass in lb_m, a = acceleration, and g_c = the gravitational constant. On earth, the value of the **mass** of an object measured in ***pounds mass*** (lb_m) is *numerically equal* to its **weight** in ***pounds force*** (lb_f). However, the student *must remember to divide* the value of m in lb_m by g_c when using this form of Newton's equation. Thus the lb_m will be divided either by 32.17 or by 386 when calculating the dynamic force. The result will be the same as when the mass is expressed in either slugs or blobs in the $F = ma$ form of the equation. Remember that in round numbers at sea level on earth

$$1\ lb_m = 1\ lb_f \qquad 1\ \text{slug} = 32.17\ lb_f \qquad 1\ \text{blob} = 386\ lb_f$$

The ***SI* system** requires that lengths be measured in meters (m), mass in kilograms (kg), and time in seconds (sec). This is sometimes also referred to as the *mks* system. Force is derived from Newton's law and the units are:

$$\text{kg m/sec}^2 = \text{newtons}$$

In the *SI* system there are distinct names for mass and force, which helps alleviate confusion.† When converting between *SI* and U.S. systems, be alert to the fact that mass converts from kilograms (kg) to either slugs (sl) or blobs (bl), and force converts from newtons (N) to pounds (lb). The gravitational constant (g_c) in the *SI* system is approximately 9.81 m/sec^2.

The *cgs* system requires that lengths be measured in centimeters (cm), mass in grams (g), and time in seconds (sec). Force is measured in dynes. The *SI* system is generally preferred over the *cgs* system.

The systems of units used in this textbook are the U.S. *ips* system and the *SI* system. Much of machine design in the United States is still done in the *ips* system, though the *SI* system is becoming more common.† Table 1-4 shows some of the variables used in this text and their units. Table 1-5 shows a number of conversion factors between commonly used units. The student is cautioned always to check the units in any equa-

* A 125-million-dollar space probe was lost because NASA failed to convert data that had been supplied in *ips* units by its contractor, Lockheed Aerospace, into the metric units used in the NASA computer programs that controlled the spacecraft. It was supposed to orbit the planet Mars, but instead either burned up in the Martian atmosphere or crashed into the planet because of this units error. *Source: The Boston Globe, October 1, 1999, p. 1.*

† A valuable resource for information on the proper use of SI units can be found at the U. S. Government NIST site at http://physics.nist.gov/cuu/Units/units.html

Another excellent resource on the proper use of metric units in machine design can be found in a pamphlet "Metric Is Simple," published and distributed by the fastener company Bossard International Inc., 235 Heritage Avenue, Portsmouth, NH 03801 http://www.bossard.com/

Table 1-4 Variables and Units

Base Units in Boldface - Abbreviations in ( )

Variable	Symbol	ips unit	fps unit	SI unit
Force	F	**pounds (lb)**	**pounds (lb)**	newtons (N)
Length	l	**inches (in)**	**feet (ft)**	**meters (m)**
Time	t	**seconds (sec)**	**seconds (sec)**	**seconds (sec)**
Mass	m	lb-sec^2/in (bl)	lb-sec^2/ft (sl)	**kilograms (kg)**
Weight	W	pounds (lb)	pounds (lb)	newtons (N)
Pressure	p	psi	psf	N/m^2 = Pa
Velocity	v	in/sec	ft/sec	m/sec
Acceleration	a	in/sec^2	ft/sec^2	m/sec^2
Stress	σ, τ	psi	psf	N/m^2 = Pa
Angle	θ	degrees (deg)	degrees (deg)	degrees (deg)
Angular velocity	ω	radians/sec	radians/sec	radians/sec
Angular acceleration	α	radians/sec^2	radians/sec^2	radians/sec^2
Torque	T	lb-in	lb-ft	N-m
Mass moment of inertia	I	lb-in-sec^2	lb-ft-sec^2	kg-m^2
Area moment of inertia	I	in^4	ft^4	m^4
Energy	E	in-lb	ft-lb	joules = N-m
Power	P	in-lb/sec	ft-lb/sec	N-m/sec = watt
Volume	V	in^3	ft^3	m^3
Specific weight	ν	lb/in^3	lb/ft^3	N/m^3
Mass density	ρ	bl/in^3	sl/ft^3	kg/m^3

tion written for a problem solution, whether in school or in professional practice. If properly written, an equation should cancel all units across the equal sign. If it does not, then you can be *absolutely sure it is **incorrect***. Unfortunately, a unit balance in an equation does not guarantee that it is correct, as many other errors are possible. Always double-check your results. You might save a life.

EXAMPLE 1-1

Units Conversion

Problem **The weight of an automobile is known in lb$_f$. Convert it to mass units in the *SI, cgs, fps*, and *ips* systems. Also convert it to lb$_m$.**

Given **The weight = 4 500 lb$_f$.**

Assumptions **The automobile is on earth at sea level.**

Table 1-5 **Selected Units Conversion Factors**

These conversion factors (and others) are built into the files UNITMAST and STUDENT

Multiply this	by	this	to get	this
acceleration				
in/sec^2	x	0.0254	=	m/sec^2
ft/sec^2	x	12	=	in/sec^2
angles				
radian	x	57.2958	=	deg
area				
in^2	x	645.16	=	mm^2
ft^2	x	144	=	in^2
area moment of inertia				
in^4	x	416 231	=	mm^4
in^4	x	4.162*E*-07	=	m^4
m^4	x	1.0*E*+12	=	mm^4
m^4	x	1.0*E*+08	=	cm^4
ft^4	x	20 736	=	in^4
density				
lb/in^3	x	27.6805	=	g/cc
lb/in^3	x	1 728	=	lb/ft^3
g/cc	x	0.001	=	g/mm^3
kg/m^3	x	1.0E-06	=	g/mm^3
force				
lb	x	4.448	=	N
N	x	1.0*E*+05	=	dyne
ton (short)	x	2 000	=	lb
length				
in	x	25.4	=	mm
ft	x	12	=	in
mass				
blob	x	386	=	lb
slug	x	32.17	=	lb
blob	x	12	=	slug
kg	x	2.205	=	lb
kg	x	9.8083	=	N
kg	x	1 000	=	g

Multiply this	by	this	to get	this
mass moment of inertia				
lb-in-sec^2	x	0.1138	=	N-m-sec^2
moments and energy				
in-lb	x	0.1138	=	N-m
ft-lb	x	12	=	in-lb
N-m	x	8.7873	=	in-lb
N-m	x	0.7323	=	ft-lb
power				
hp	x	550	=	ft-lb/sec
hp	x	33 000	=	ft-lb/min
hp	x	6 600	=	in-lb/sec
hp	x	745.7	=	watts
N-m/sec	x	8.7873	=	in-lb/sec
pressure and stress				
psi	x	6 894.8	=	Pa
psi	x	6.895*E*-3	=	MPa
psi	x	144	=	psf
kpsi	x	1 000	=	psi
N/m^2	x	1	=	Pa
N/mm^2	x	1	=	MPa
spring rate				
lb/in	x	175.126	=	N/m
lb/ft	x	0.08333	=	lb/in
stress intensity				
MPa-m$^{0.5}$	x	0.909	=	kpsi-in$^{0.5}$
velocity				
in/sec	x	0.0254	=	m/sec
ft/sec	x	12	=	in/sec
rad/sec	x	9.5493	=	rpm
volume				
in^3	x	16 387.2	=	mm^3
ft^3	x	1 728	=	in^3
cm^3	x	0.061023	=	in^3
m^3	x	1.0*E*+9	=	mm^3

Solution

1 Equation 1.4*a* (p. 22) is valid for the first four systems listed.

For the *fps* system:

$$m = \frac{W}{g} = \frac{4\,500 \text{ lb}_f}{32.17 \text{ ft/sec}^2} = 139.9 \frac{\text{lb}_f - \text{sec}^2}{\text{ft}} = 139.9 \text{ slugs} \tag{a}$$

For the *ips* system:

$$m = \frac{W}{g} = \frac{4\,500 \text{ lb}_f}{386 \text{ in/sec}^2} = 11.66 \frac{\text{lb}_f - \text{sec}^2}{\text{in}} = 11.66 \text{ blobs} \tag{b}$$

For the *SI* system:

$$W = 4\,500 \text{ lb} \frac{4.448 \text{ N}}{\text{lb}} = 20\,016 \text{ N}$$

$$m = \frac{W}{g} = \frac{20\,016 \text{ N}}{9.81 \text{ m/sec}^2} = 2040 \frac{\text{N} - \text{sec}^2}{\text{m}} = 2\,040 \text{ kg} \tag{c}$$

For the *cgs* system:

$$W = 4\,500 \text{ lb} \frac{4.448E5 \text{ dynes}}{\text{lb}} = 2.002E9 \text{ dynes}$$

$$m = \frac{W}{g} = \frac{2.002E9 \text{ dynes}}{981 \text{ cm/sec}^2} = 2.04E6 \frac{\text{dynes} - \text{sec}^2}{\text{cm}} = 2.04E6 \text{ g} \tag{d}$$

2 For mass expressed in lb_m, equation 1.4*b* (p. 22) must be used.

$$m = W\frac{g_c}{g} = 4\,500 \text{ lb}_f \frac{386 \text{ in/sec}^2}{386 \text{ in/sec}^2} = 4\,500 \text{ lb}_m \tag{e}$$

Note that lb_m is numerically equal to lb_f and so must not be used as a mass unit unless you are using the form of Newton's law expressed as equation 1.4*b*.

1.10 SUMMARY

Design can be fun and frustrating at the same time. Because design problems are very unstructured, a large part of the task is creating sufficient structure to make it solvable. This naturally leads to multiple solutions. To students used to seeking an answer that matches the one in the "back of the book" this exercise can be frustrating. There is no "one right answer" to a design problem, only answers that are arguably better or worse than others. The marketplace has many examples of this phenomenon. How many different makes and models of new automobiles are available? Don't they all do more or less the same task? But you probably have your own opinion about which ones do the task better than others. Moreover, the task definition is not exactly the same for all examples. A four-wheel-drive automobile is designed for a slightly different problem definition than is a two-seat sports car (though some examples incorporate both those features).

The message to the beginning designer then is to be open-minded about the design problems posed. Don't approach design problems with the attitude of trying to find "the right answer," as there is none. Rather, be daring! Try something radical. Then test it with analysis. When you find it doesn't work, don't be disappointed; instead realize that you have learned something about the problem you didn't know before. Negative results are still results! We learn from our mistakes and can then design a better solution the next time. This is why *iteration* is so crucial to successful design.

The computer is a necessary tool to the solution of contemporary engineering problems. Problems can be solved more quickly and more accurately with proper use of computer-aided engineering (CAE) software. However, the results are only as good as the quality of the engineering models and data used. The engineer should not rely on computer-generated solutions without also developing and applying a thorough understanding of the fundamentals on which the model and the CAE tools are based.

Important Equations Used in This Chapter

See the referenced sections for information on the proper use of these equations.

Mass (see Section 1.9):

$$m = \frac{W}{g_c} \tag{1.3}$$

Dynamic Force—for use with standard mass units (kg, slugs, blobs) (see Section 1.9):

$$F = ma \tag{1.4a}$$

Dynamic Force—for use with mass in $lb_m = lb_f$ (see Section 1.9):

$$F = \frac{ma}{g_c} \tag{1.4b}$$

1.11 REFERENCES

1 *Random House Dictionary of the English Language*. 2nd ed. unabridged, S.B. Flexner, ed., Random House: New York, 1987, p. 1151.

2 **R. L. Norton**, *Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines*, 3ed. McGraw-Hill: New York, 2004, pp. 7-14.

3 **Autocad**, Autodesk Inc., http://usa.autodesk.com

4 **Unigraphics**, EDS, Cyprus, CA, http://www.eds.com

5 **ADAMS**, Mechanical Dynamics, MSC Software, http://www.krev.com

6 **Working Model**, MSC Software, http://www.krev.com.

7 **Pro/Engineer**, Parametric Technology Corp., Waltham, MA, http://www.ptc.com

8 **TK Solver**, Universal Technical Systems, Rockford, IL, http://www.uts.com

9 **Mathcad**, Mathsoft Inc., Cambridge, MA, http://www.mathsoft.com

10 **Excel**, Microsoft Corp., Redmond, WA, http://www.microsoft.com

11 **MATLAB**, Mathworks Inc., Natick, MA, http://www.mathworks.com

12 **Solidworks**, Solidworks Corp., Concord, MA, http://www.solidworks.com

1.12 WEB REFERENCES

http://www.onlineconversion.com

Convert just about anything to anything else. Over 5,000 units, and 50,000 conversions.

http://www.katmarsoftware.com/uconeer.htm

Download a free units converter program for engineers.

http://global.ihs.com

Search a collection of technical standards with over 500,000 documents available for electronic download.

http://www.thomasnet.com

An online resource for finding companies and products manufactured in North America.

1.13 BIBLIOGRAPHY

For information on creativity and the design process, the following are recommended:

J. L. Adams, *The Care and Feeding of Ideas*. 3rd ed. Addison Wesley: Reading, Mass., 1986.

J. L. Adams, *Conceptual Blockbusting*. 3rd ed. Addison Wesley: Reading, Mass., 1986.

J. R. M. Alger and C. V. Hays, *Creative Synthesis in Design*. Prentice-Hall: Englewood Cliffs, N.J., 1964.

M. S. Allen, *Morphological Creativity*. Prentice-Hall: Englewood Cliffs, N.J., 1962.

H. R. Buhl, *Creative Engineering Design*. Iowa State University Press: Ames, Iowa, 1960.

W. J. J. Gordon, *Synectics*. Harper and Row: New York, 1962.

J. W. Haefele, *Creativity and Innovation*. Reinhold: New York, 1962.

L. Harrisberger, *Engineersmanship*. 2nd ed. Brooks/Cole: Monterey, Calif., 1982.

D. A. Norman, *The Psychology of Everyday Things*. Basic Books: New York, 1986.

A. F. Osborne, *Applied Imagination*. Scribners: New York, 1963.

C. W. Taylor, *Widening Horizons in Creativity*. John Wiley: New York, 1964.

E. K. Von Fange, *Professional Creativity*. Prentice-Hall: Englewood Cliffs, N.J., 1959.

For information on writing engineering reports, the following are recommended:

R. Barrass, *Scientists Must Write*. Chapman and Hall: New York, 1978.

W. G. Crouch and R. L. Zetler, *A Guide to Technical Writing*. 3rd ed. The Ronald Press Co.: New York, 1964.

D. S. Davis, *Elements of Engineering Reports*. Chemical Publishing Co.: New York, 1963.

D. E. Gray, *So You Have to Write a Technical Report*. Information Resources Press: Washington, D.C., 1970.

H. B. Michaelson, *How to Write and Publish Engineering Papers and Reports*. ISI: Philadelphia, Pa., 1982.

J. R. Nelson, *Writing the Technical Report*. 3rd ed. McGraw-Hill: New York, 1952.

1

Table P1-0
Topic/Problem Matrix

1.4 Engineering Model	1-1, 1-2, 1-3
1.9 Units	1-4, 1-5, 1-6, 1-7, 1-8

1.14 PROBLEMS

1-1 It is often said, "*Build a better mousetrap and the world will beat a path to your door.*" Consider this problem and write a goal statement and a set of at least 12 task specifications that you would apply to its solution. Then suggest 3 possible concepts to achieve the goal. Make annotated, freehand sketches of the concepts.

1-2 A bowling machine is desired to allow quadriplegic youths, who can only move a joystick, to engage in the sport of bowling at a conventional bowling alley. Consider the factors involved, write a goal statement, and develop a set of at least 12 task specifications that constrain this problem. Then suggest 3 possible concepts to achieve the goal. Make annotated, freehand sketches of the concepts.

1-3 A quadriplegic needs an automated page-turner to allow her to read books without assistance. Consider the factors involved, write a goal statement, and develop a set of at least 12 task specifications that constrain this problem. Then suggest 3 possible concepts to achieve the goal. Make annotated, freehand sketches of the concepts.

*1-4 Convert a mass of 1000 lb_m to (a) lb_f, (b) slugs, (c) blobs, (d) kg.

*1-5 A 250-lb_m mass is accelerated at 40 in/sec^2. Find the force in lb needed for this acceleration.

*1-6 Express a 100 kg mass in units of slugs, blobs, and lb_m. How much does this mass weigh in lb_f and in N?

1-7 Prepare an interactive computer program (using, for example, *Excel, Mathcad, MATLAB,* or *TK Solver*) from which the cross-sectional properties for the shapes shown on the inside front cover can be calculated. Arrange the program to deal with both *ips* and *SI* units systems and convert the results between those systems.

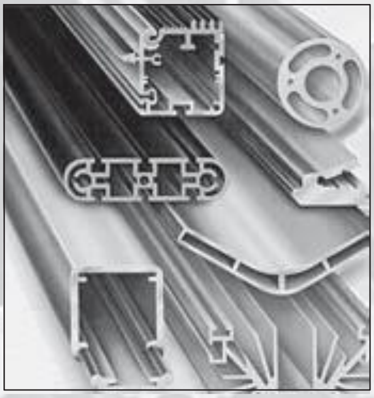
1-8 Prepare an interactive computer program (using, for example, *Excel, Mathcad, MATLAB,* or *TK Solver*) from which the mass properties for the solids shown on the page opposite the inside front cover can be calculated. Arrange the program to deal with both *ips* and *SI* units systems and convert the results between those systems.

1-9 Convert the program written for Problem 1-7 to have and use a set of functions or subroutines that can be called from within any program in that language to solve for the cross-sectional properties of the shapes shown on the inside front cover.

1-10 Convert the program written for Problem 1-8 to have and use a set of functions or subroutines that can be called from within any program in that language to solve for the mass properties for the solids shown on the page opposite the inside front cover.

* Answers to these problems are provided in Appendix D.

2



MATERIALS AND PROCESSES

There is no subject so old that something new cannot be said about it.

Dostoevsky

2.0 INTRODUCTION

Whatever you design, you must make it out of some material and be able to manufacture it. A thorough understanding of material properties, treatments, and manufacturing processes is essential to good machine design. It is assumed that the reader has had a first course in material science. This chapter presents a brief review of some basic metallurgical concepts and a short summary of engineering material properties to serve as background for what follows. This is not intended as a substitute for a text on material science, and the reader is encouraged to review references such as those listed in the bibliography of this chapter for more detailed information. Later chapters of this text will explore some of the common material-failure modes in more detail.

Table 2-0 shows the variables used in this chapter and references the equations, figures, or sections in which they are used. At the end of the chapter, a summary section is provided which groups the significant equations from this chapter for easy reference and identifies the chapter section in which they are discussed.

2.1 MATERIAL-PROPERTY DEFINITIONS

Mechanical properties of a material are generally determined through destructive testing of samples under controlled loading conditions. The test loadings do not accurately duplicate actual service loadings experienced by machine parts except in certain special cases. Also, there is no guarantee that the particular piece of material you purchase for your part will exhibit the same strength properties as the samples of similar materials tested previously. There will be some statistical variation in the strength of any par-

Table 2-0 Variables Used in This Chapter

Symbol	Variable	ips units	SI units	See
A	area	in^2	m^2	Sect. 2.1
A_0	original area, test specimen	in^2	m^2	Eq. 2.1*a*
E	Young's modulus	psi	Pa	Eq. 2.2
el	elastic limit	psi	Pa	Figure 2-2
f	fracture point	none	none	Figure 2-2
G	shear modulus, modulus of rigidity	psi	Pa	Eq. 2.4
HB	Brinell hardness	none	none	Eq. 2.10
HRB	Rockwell B hardness	none	none	Sect. 2.4
HRC	Rockwell C hardness	none	none	Sect. 2.4
HV	Vickers hardness	none	none	Sect. 2.4
J	polar second moment of area	in^4	m^4	Eq. 2.5
K	stress intensity	kpsi-in$^{0.5}$	MPa-m$^{0.5}$	Sect. 2.1
K_c	fracture toughness	kpsi-in$^{0.5}$	MPa-m$^{0.5}$	Sect. 2.1
l_0	gage length, test specimen	in	m	Eq. 2.3
N	number of cycles	none	none	Figure 2-10
P	force or load	lb	N	Sect. 2.1
pl	proportional limit	psi	Pa	Figure 2-2
r	radius	in	m	Eq. 2.5*a*
S_d	standard deviation	any	any	Eq. 2.9
S_e	endurance limit	psi	Pa	Figure 2-10
S_{el}	strength at elastic limit	psi	Pa	Eq. 2.7
S_f	fatigue strength	psi	Pa	Figure 2-10
S_{us}	ultimate shear strength	psi	Pa	Eq. 2.5
S_{ut}	ultimate tensile strength	psi	Pa	Figure 2-2
S_y	tensile yield strength	psi	Pa	Figure 2-2
S_{ys}	shear yield strength	psi	Pa	Eq. 2.5c
T	torque	lb-in	N-m	Sect. 2.1
U_R	modulus of resilience	psi	Pa	Eq. 2.7
U_T	modulus of toughness	psi	Pa	Eq. 2.8
y	yield point	none	none	Figure 2-2
ε	strain	none	none	Eq. 2.1*b*
σ	tensile stress	psi	Pa	Sect. 2.1
τ	shear stress	psi	Pa	Eq. 2.3
θ	angular deflection	rad	rad	Eq. 2.3
μ	arithmetic mean value	any	any	Eq. 2.9*b*
ν	Poisson's ratio	none	none	Eq. 2.4

ticular sample compared to the average tested properties for that material. For this reason, many of the published strength data are given as minimum values. It is with these caveats that we must view all published material-property data, as it is the engineer's responsibility to ensure the safety of his or her design.

The best material-property data will be obtained from destructive or nondestructive testing under actual service loadings of prototypes of your actual design, made from the actual materials by the actual manufacturing process. This is typically done only when the economic and safety risks are high. Manufacturers of aircraft, automobiles, motorcycles, snowmobiles, farm equipment, and other products regularly instrument and test finished assemblies under real or simulated service conditions.

In the absence of such specific test data, the engineer must adapt and apply published material-property data from standard tests to the particular situation. The *American Society for Testing and Materials* (ASTM) defines standards for test specimens and test procedures for a variety of material-property measurements.* The most common material test used is the tensile test.

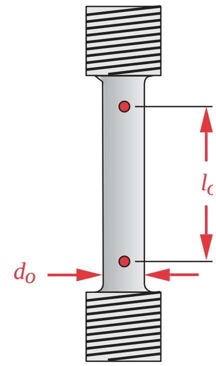


FIGURE 2-1

A Tensile Test Specimen

The Tensile Test

A typical tensile test specimen is shown in Figure 2-1. This tensile bar is machined from the material to be tested in one of several standard diameters d_o and gage lengths l_o. The gage length is an arbitrary length defined along the small-diameter portion of the specimen by two indentations so that its increase can be measured during the test. The larger-diameter ends of the bar are threaded for insertion into a tensile test machine which is capable of applying either controlled loads or controlled deflections to the ends of the bar, and the gage-length portion is mirror polished to eliminate stress concentrations from surface defects. The bar is stretched slowly in tension until it breaks, while the load and the distance across the gage length (or alternatively the strain) are continuously monitored. The result is a stress-strain plot of the material's behavior under load as shown in Figure 2-2*a*, which depicts a curve for a low-carbon or "mild" steel.

Stress and Strain Note that the parameters measured are load and deflection, but those plotted are stress and strain. **Stress** (σ) is defined as *load per unit area* (or *unit load*) and for the tensile specimen is calculated from

$$\sigma = \frac{P}{A_o} \tag{2.1a}$$

where P is the applied load at any instant and A_o is the original cross-sectional area of the specimen. The stress is assumed to be uniformly distributed across the cross section. The stress units are psi or Pa.

Strain is the *change in length per unit length* and is calculated from

$$\varepsilon = \frac{l - l_o}{l_o} \tag{2.1b}$$

where l_o is the original gage length and l is the gage length at any load P. The strain is unitless, being length divided by length.

Modulus of Elasticity This tensile stress-strain curve provides us with a number of useful material parameters. Point *pl* in Figure 2-2*a* is the **proportional limit** below which the stress is proportional to the strain, as expressed by the one-dimensional form of **Hooke's law**:

$$E = \frac{\sigma}{\varepsilon} \tag{2.2}$$

* *ASTM, 1994 Annual Book of ASTM Standards*, Vol. 03.01, Am. Soc. for Testing and Materials, Philadelphia, PA.

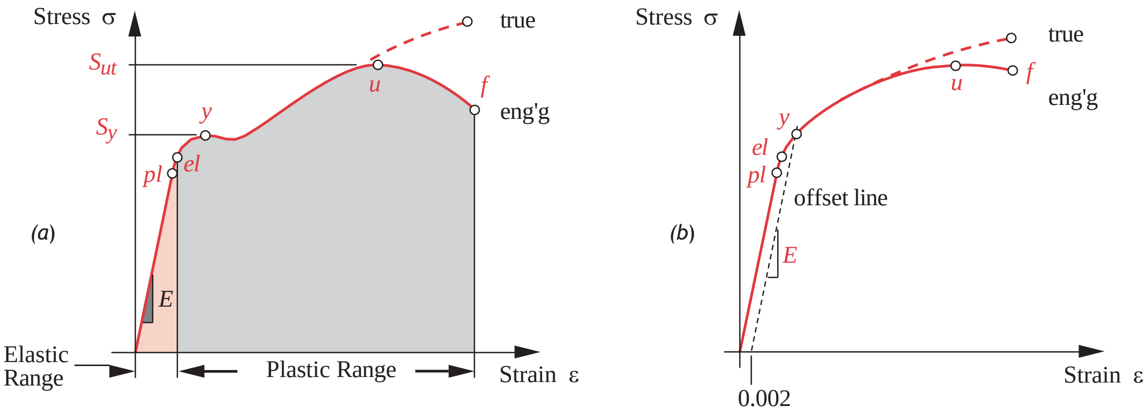


FIGURE 2-2

Engineering and True Stress-Strain Curves for Ductile Materials: (*a*) Low-Carbon Steel (*b*) Annealed High-Carbon Steel

where E defines the slope of the stress-strain curve up to the proportional limit and is called **Young's modulus** or the **modulus of elasticity** of the material. E is a measure of the stiffness of the material in its elastic range and has the units of stress. Most metals exhibit this linear stiffness behavior and also have elastic moduli that vary very little with heat treatment or with the addition of alloying elements. For example, the highest-strength steel has the same E as the lowest-strength steel at about 30 Mpsi (207 GPa). For most ductile materials (defined below), the modulus of elasticity in compression is the same as in tension. This is not true for cast irons and other brittle materials (defined below) or for magnesium.

ELASTIC LIMIT The point labeled *el* in Figure 2-2*a* is the **elastic limit**, or the point beyond which the material will take a permanent set, or plastic deformation. The elastic limit marks the boundary between the **elastic-behavior** and **plastic-behavior** regions of the material. Points *el* and *pl* are typically so close together that they are often considered to be the same.

YIELD STRENGTH At a point y slightly above the elastic limit, the material begins to yield more readily to the applied stress, and its rate of deformation increases (note the lower slope). This is called the **yield point**, and the value of stress at that point defines the **yield strength** S_y of the material.

Materials that are very ductile, such as low-carbon steels, will sometimes show an apparent drop in stress just beyond the yield point, as shown in Figure 2-2*a*. Many less ductile materials, such as aluminum and medium- to high-carbon steels, will not exhibit this apparent drop in stress and will look more like Figure 2-2*b*. The yield strength of a material that does not exhibit a clear yield point has to be defined with an offset line, drawn parallel to the elastic curve and offset some small percentage along the strain axis. An offset of 0.2% strain is most often used. The yield strength is then taken at the intersection of the stress-strain curve and the offset line as shown in Figure 2-2*b*.

ULTIMATE TENSILE STRENGTH The stress in the specimen continues to increase nonlinearly to a peak or **ultimate tensile strength** value S_{ut} at point u. This is considered to be the largest tensile stress the material can sustain before breaking. However, for

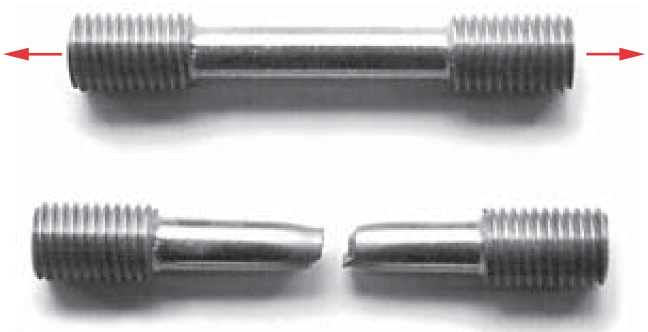


FIGURE 2-3

A Tensile Test Specimen of Mild, Ductile Steel Before and After Fracture

the ductile steel curve shown, the stress appears to fall off to a smaller value at the fracture point *f*. The drop in apparent stress before the fracture point (from *u* to *f* in Figure 2-2*a*) is an artifact caused by the "necking-down" or reduction in area of the ductile specimen. The reduction of cross-sectional area is nonuniform along the length of the specimen as can be seen in Figure 2-3.

Because the stress is calculated using the original area A_o in equation 2.1*a*, it understates the true value of stress after point *u*. It is difficult to accurately monitor the dynamic change in cross-sectional area during the test, so these errors are accepted. The strengths of different materials can still be compared on this basis. When based on the uncorrected area A_o this is called the **engineering stress-strain curve,** as shown in Figure 2-2.

The stress at fracture is actually larger than shown. Figure 2-2 also shows the **true stress-strain curve** that would result if the change in area were accounted for. The **engineering stress-strain** data from Figure 2-2 are typically used in practice. The most commonly used strength values for **static loading** are the yield strength S_y and the ultimate tensile strength S_{ut}. The material stiffness is defined by Young's modulus, E.

In comparing the properties of different materials, it is quite useful to express those properties normalized to the material's density. Since light weight is nearly always a goal in design, we seek the lightest material that has sufficient strength and stiffness to withstand the applied loads. The **specific strength** of a material is defined as *the strength divided by the density*. Unless otherwise specified, strength in this case is assumed to mean ultimate tensile strength, though any strength criterion can be so normalized. The **strength-to-weight ratio** (SWR) is another way to express the specific strength. **Specific stiffness** is the *Young's modulus divided by material density*.

Ductility and Brittleness

The tendency for a material to deform significantly before fracturing is a measure of its ductility. The absence of significant deformation before fracture is called brittleness.

DUCTILITY The stress-strain curve in Figure 2-2*a* is of a ductile material, mild steel. Take a common paper clip made of mild-steel wire. Straighten it out with your fingers.

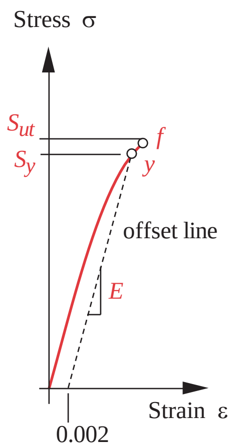


FIGURE 2-4

Stress-Strain Curve of a Brittle Material

FIGURE 2-5

A Tensile Test Specimen of Brittle Cast Iron Before and After Fracture

Bend it into some new shape. You are yielding this ductile steel wire but not fracturing it. You are operating between point *y* and point *f* on the stress-strain curve of Figure 2-2*a*. The presence of a significant plastic region on the stress-strain curve is evidence of ductility.

Figure 2-3 shows a test specimen of ductile steel after fracture. The distortion called *necking-down* can clearly be seen at the break. The fracture surface appears torn and is laced with hills and valleys, also indicating a ductile failure. The **ductility** of a material is measured by its percent elongation to fracture, or percent reduction in area at fracture. Materials with more than 5% elongation at fracture are considered ductile.

BRITTLENESS Figure 2-4 shows a stress-strain curve for a brittle material. Note the lack of a clearly defined yield point and the absence of any plastic range before fracture. Repeat your paper-clip experiment, this time using a wooden toothpick or matchstick. Any attempt to bend it results in fracture. Wood is a brittle material.

Brittle materials do not exhibit a clear yield point, so the yield strength has to be defined at the intersection of the stress-strain curve and an offset line, drawn parallel to the elastic curve and offset some small percentage such as 0.2% along the strain axis. Some brittle materials like cast iron do not have a linear elastic region, and the offset line is taken at the average slope of the region. Figure 2-5 shows a cast iron test specimen after fracture. The break shows no evidence of necking and has the finer surface contours typical of a brittle fracture.

The same metals can be either ductile or brittle depending on the way they are manufactured, worked, and heat treated. Metals that are **wrought** (meaning drawn or pressed into shape in a solid form while either hot or cold) can be more ductile than metals that are cast by pouring molten metal into a mold or form. There are many exceptions to this broad statement, however. The cold working of metal (discussed below) tends to reduce its ductility and increase its brittleness. Heat treatment (discussed below) also has a marked effect on the ductility of steels. Thus it is difficult to generalize about the relative ductility or brittleness of various materials. A careful look at all the mechanical properties of a given material will tell the story.

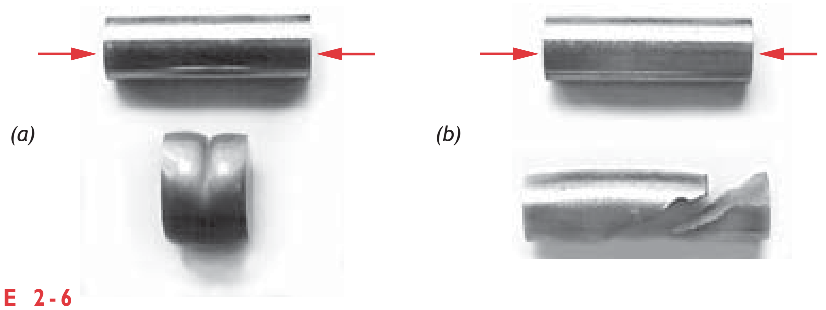


FIGURE 2-6

Compression Test Specimens Before and After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

The Compression Test

The tensile test machine can be run in reverse to apply a compressive load to a specimen that is a constant-diameter cylinder as shown in Figure 2-6. It is difficult to obtain a useful stress-strain curve from this test because a ductile material will yield and increase its cross-sectional area, as shown in Figure 2-6*a*, eventually stalling the test machine. The ductile sample will not fracture in compression. If enough force were available from the machine, it could be crushed into a pancake shape. Most ductile materials have compressive strengths similar to their tensile strengths, and the tensile stress-strain curve is used to represent their compressive behavior as well. A material that has essentially equal tensile and compressive strengths is called an **even material**.

Brittle materials will fracture when compressed. A failed specimen of brittle cast iron is shown in Figure 2-6*b*. Note the rough, angled fracture surface. The reason for the failure on an angled plane is discussed in Chapter 4. Brittle materials generally have much greater strength in compression than in tension. Compressive stress-strain curves can be generated, since the material fractures rather than crushes and the cross-sectional area doesn't change appreciably. A material that has different tensile and compressive strengths is called an **uneven material**.

The Bending Test

A thin rod, as shown in Figure 2-7, is simply supported at each end as a beam and loaded transversely in the center of its length until it fails. If the material is ductile, failure is by yielding, as shown in Figure 2-7*a*. If the material is brittle, the beam fractures as shown in Figure 2-7*b*. Stress-strain curves are not generated from this test because the stress distribution across the cross section is not uniform. The tensile test's σ–ε curve is used to predict failure in bending, since the bending stresses are tensile on the convex side and compressive on the concave side of the beam.

The Torsion Test

The shear properties of a material are more difficult to determine than its tensile properties. A specimen similar to the tensile test specimen is made with noncircular details on its ends so that it can be twisted axially to failure. Figure 2-8 shows two such samples, one of ductile steel and one of brittle cast iron. Note the painted lines along

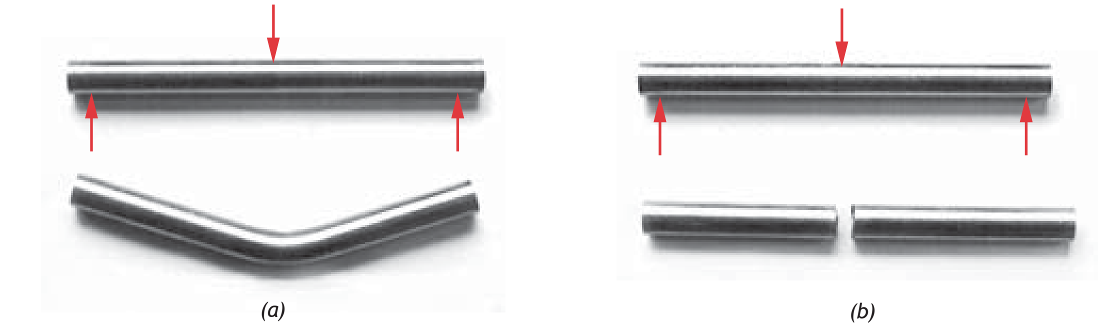


FIGURE 2-7

Bending Test Specimens Before and After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

their lengths. The lines were originally straight in both cases. The helical twist in the ductile specimen's line after failure shows that it wound up for several revolutions before breaking. The brittle, torsion-test specimen's line is still straight after failure as there was no significant plastic distortion before fracture.

Table 2-1

Poisson's Ratio ν

Material	ν
Aluminum	0.34
Copper	0.35
Iron	0.28
Steel	0.28
Magnesium	0.33
Titanium	0.34

MODULUS OF RIGIDITY The stress-strain relation for pure torsion is defined by

$$\tau = \frac{Gr\theta}{l_o} \tag{2.3}$$

where τ is the shear stress, r is the radius of the specimen, l_o is the gage length, θ is the angular twist in radians, and G is the shear modulus or **modulus of rigidity**. G can be defined in terms of Young's modulus E and Poisson's ratio ν:

$$G = \frac{E}{2(1+\nu)} \tag{2.4}$$

Poisson's ratio (ν) is the ratio between lateral and longitudinal strain and for most metals is around 0.3 as shown in Table 2-1.

ULTIMATE SHEAR STRENGTH The breaking strength in torsion is called the ultimate shear strength or modulus of rupture S_{us} and is calculated from

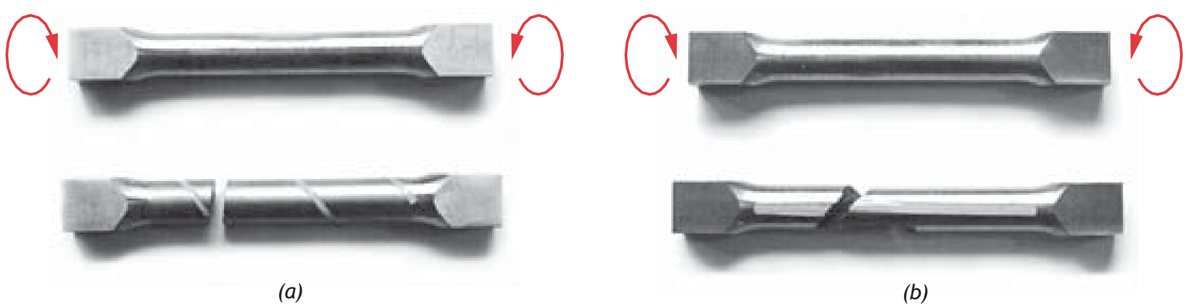


FIGURE 2-8

Torsion Test Specimens Before and After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

$$S_{us} = \frac{Tr}{J} \tag{2.5a}$$

where T is the applied torque necessary to break the specimen, r is the radius of the specimen, and J is the polar second moment of area of the cross section. The distribution of stress across the section loaded in torsion is not uniform. It is zero at the center and maximum at the outer radius. Thus the outer portions have already plastically yielded while the inner portions are still below the yield point. This nonuniform stress distribution in the torsion test (unlike the uniform distribution in the tension test) is the reason for calling the measured value at failure of a solid bar in torsion a *modulus of rupture*. A thin-walled tube is a better torsion-test specimen than a solid bar for this reason and can give a better measure of the ultimate shear strength.

In the absence of available data for the ultimate shear strength of a material, a reasonable approximation can be obtained from tension test data:*

$$\begin{aligned} &\text{steels:} && S_{us} \cong 0.80 S_{ut} \\ &\text{other ductile metals:} && S_{us} \cong 0.75 S_{ut} \end{aligned} \tag{2.5b}$$

Note that the shear yield strength has a different relationship to the tensile yield strength:

$$S_{ys} \cong 0.577 S_y \tag{2.5c}$$

This relationship is derived in Chapter 5, where failure of materials under static loading is discussed in more detail.

Fatigue Strength and Endurance Limit

The tensile test and the torsion test both apply loads slowly and only once to the specimen. These are static tests and measure static strengths. While some machine parts may see only static loads in their lifetime, most will see loads and stresses that vary with time. Materials behave very differently in response to loads that come and go (called **fatigue loads**) than they do to loads that remain static. Most of machine design deals with the design of parts for time-varying loads, so we need to know the **fatigue strength** of materials under these loading conditions.

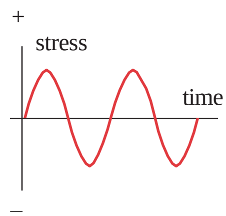


FIGURE 2-9
Time-Varying Loading

One test for fatigue strength is the R. R. Moore rotating-beam test in which a similar, but slightly smaller, test specimen than that shown in Figure 2-1 is loaded as a beam in bending while being rotated by a motor. Recall from your first course in strength of materials that a bending load causes tension on one side of a beam and compression on the other. (See Sections 4-9 and 4-10 for a review of beams in bending.) The rotation of the beam causes any one point on the surface to go from compression to tension to compression each cycle. This creates a load-time curve as shown in Figure 2-9.

The test is continued at a particular stress level until the part fractures, and the number of cycles N is then noted. Many samples of the same material are tested at various stress levels S until a curve similar to Figure 2-10 is generated. This is called a Wohler strength-life diagram or an S-N diagram. It depicts the breaking strength of a particular material at various numbers of repeated cycles of fully reversed stress.

* In Chapter 14 on helical spring design, an empirical relationship for the ultimate shear strength of small diameter steel wire, based on extensive testing of wire in torsion, is presented in equation 14.4 (p. 793) and is S_{us} = 0.67 S_{ut}. This is obviously different than the general approximation for steel in equation 2.5b. The best data for material properties will always be obtained from tests of the same material, geometry, and loading as the part will be subjected to in service. In the absence of direct test data we must rely on approximations of the sort in equation 2.5b and apply suitable safety factors based on the uncertainty of these approximations.

2

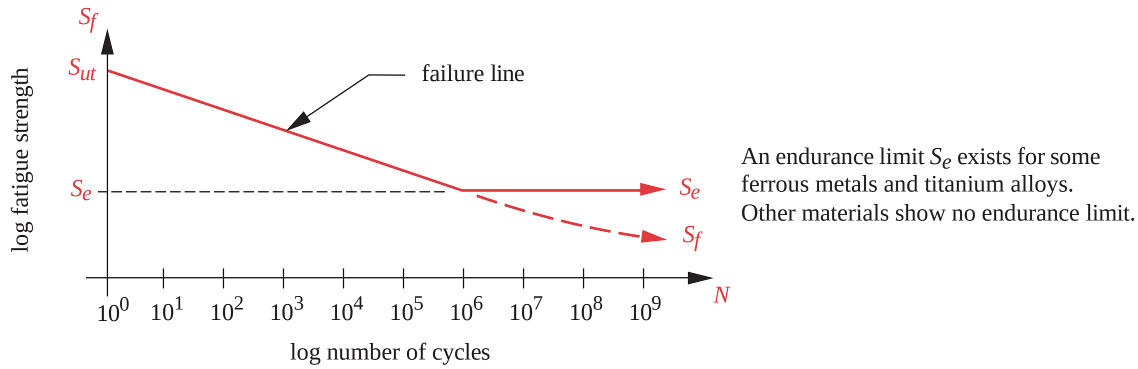


FIGURE 2-10

Wohler Strength-Life or *S-N* Diagram Plots Fatigue Strength Against Number of Fully Reversed Stress Cycles

Note in Figure 2-10 that the **fatigue strength** S_f at one cycle is the same as the static strength S_{ut}, and it decreases steadily with increasing numbers of cycles N (on a log-log plot) until reaching a plateau at about 10^6 cycles. This plateau in fatigue strength exists only for certain metals (notably steels and some titanium alloys) and is called the endurance limit S_e. Fatigue strengths of other materials keep falling beyond that point. While there is considerable variation among materials, their raw (or uncorrected) fatigue strengths at about $N = 10^6$ cycles tend to be no more than about 40–50% of their static tensile strength S_{ut}. This is a significant reduction and, as we will learn in Chapter 6, further reductions in the fatigue strengths of materials will be necessary due to other factors such as surface finish and type of loading.

It is important at this stage to remember that the tensile stress-strain test does not tell the whole story and that a material's static strength properties are seldom adequate by themselves to predict failure in a machine-design application. This topic of fatigue strength and endurance limit is so important and fundamental to machine design that we devote Chapter 6 exclusively to a study of fatigue failure.

The rotating-beam test is now being supplanted by axial-tension tests performed on modern test machines which can apply time-varying loads of any desired character to the axial-test specimen. This approach provides more testing flexibility and more accurate data because of the uniform stress distribution in the tensile specimen. The results are consistent with (but slightly lower-valued than) the historical rotating-beam test data for the same materials.

Impact Resistance

The stress-strain test is done at very low, controlled strain rates, allowing the material to accommodate itself to the changing load. If the load is suddenly applied, the energy absorption capacity of the material becomes important. The energy in the differential element is its **strain energy density** (strain energy per unit volume U_0), or the area under the stress-strain curve at any particular strain.

$$U_0 = \int_0^{\varepsilon} \sigma \, d\varepsilon \tag{2.6a}$$

The strain energy U is equal to the strain energy density integrated over the volume v.

$$U = \int_v U_0 \, dv \tag{2.6b}$$

The **resilience** and **toughness** of the material are measures, respectively, of the strain energy present in the material at the elastic limit or at the fracture point.

RESILIENCE The ability of a material to absorb energy per unit volume without permanent deformation is called its **resilience** U_R (also called **modulus of resilience**) and is equal to the area under the stress-strain curve up to the elastic limit, shown as the color-shaded area in Figure 2-2*a*. Resilience is defined as:

$$\begin{aligned} U_R &= \int_0^{\varepsilon_{el}} \sigma d\varepsilon = \frac{1}{2} S_{el}\varepsilon_{el} \\ &= \frac{1}{2} S_{el} \frac{S_{el}}{E} = \frac{1}{2}\frac{S_{el}^2}{E} \\ U_R &\cong \frac{1}{2}\frac{S_y^2}{E} \end{aligned} \tag{2.7}$$

where S_{el} and ε_{el} represent, respectively, the strength and strain at the elastic limit. Substitution of Hooke's law from equation 2.2 expresses the relationship in terms of strength and Young's modulus. Since the S_{el} value is seldom available, a reasonable approximation of resilience can be obtained by using the yield strength S_y instead.

This relationship shows that a stiffer material of the same elastic strength is less resilient than a more compliant one. A rubber ball can absorb more energy without permanent deformation than one made of glass.

TOUGHNESS The ability of a material to absorb energy per unit volume without fracture is called its **toughness** U_T (also called **modulus of toughness**) and is equal to the area under the stress-strain curve up to the fracture point, shown as the entire shaded area in Figure 2-2*a*. Toughness is defined as:

$$\begin{aligned} U_T &= \int_0^{\varepsilon_f} \sigma d\varepsilon \\ &\cong \left(\frac{S_y + S_{ut}}{2}\right)\varepsilon_f \end{aligned} \tag{2.8}$$

where S_{ut} and ε_f represent, respectively, the ultimate tensile strength and the strain at fracture. Since an analytical expression for the stress-strain curve is seldom available for actual integration, an approximation of toughness can be obtained by using the average of the yield and ultimate strengths and the strain at fracture to calculate an area. The units of toughness and resilience are energy per unit volume (in-lb/in^3 or joules/m^3). Note that these units are numerically equivalent to psi or Pa.

A ductile material of similar ultimate strength to a brittle one will be much more tough. A sheet-metal automobile body will absorb more energy from a collision through plastic deformation than will a brittle, fiberglass body.*

* It is interesting to note that one of the toughest and strongest materials known is that of *spider webs*! These tiny arachnids spin a monofilament that has an ultimate tensile strength of 200 to 300 kpsi (1380 to 2070 MPa) and 35% elongation to fracture! It also can absorb more energy without rupture than any fiber known, absorbing 3 times as much energy as *Kevlar*, the man-made fiber used for bullet-proof vests. According to the *Boston Globe*, January 18, 2002, researchers in Canada and the U.S. have synthesized a material with similar properties to spider silk in strands up to 10-ft long with strengths of 1/4 to 1/3 that of natural silk fiber, "stronger than a steel wire of similar weight," and one that has greater elasticity than organic silk fiber.

Impact Testing Various tests have been devised to measure the ability of materials to withstand impact loading. The **Izod** and the **Charpy** tests are two such procedures which involve striking a notched specimen with a pendulum and recording the kinetic energy needed to break the specimen at a particular temperature. While these data do not directly correlate with the area under the stress-strain curve, they nevertheless provide a means to compare the energy absorption capacity of various materials under controlled conditions. Materials handbooks such as those listed in this chapter's bibliography give data on the impact resistance of various materials.

Fracture Toughness

Fracture toughness K_c (not to be confused with the modulus of toughness defined above) is *a material property that defines its ability to resist stress at the tip of a crack*. The fracture toughness of a material is measured by subjecting a standardized, precracked test specimen to cyclical tensile loads until it breaks. Cracks create very high local stress concentrations which cause local yielding (see Section 4.15). The effect of the crack on the local stress is measured by a **stress intensity factor** K which is defined in Section 5.3. When the stress intensity K reaches the fracture toughness K_c, a sudden fracture occurs with no warning. The study of this failure phenomenon is called **fracture mechanics** and it is discussed in more detail in Chapters 5 and 6.

Creep and Temperature Effects

The tensile test, while slow, does not last long compared to the length of time an actual machine part may be subjected to constant loading. All materials will, under the right environmental conditions (particularly elevated temperatures), slowly creep (deform) under stress loadings well below the level (yield point) deemed safe in the tensile test. Ferrous metals tend to have negligible creep at room temperature or below. Their creep rates increase with increasing ambient temperature, usually becoming significant around 30–60% of the material's absolute melting temperature.

Low-melt-temperature metals such as lead, and many polymers, can exhibit significant creep at room temperature as well as increasing creep rates at higher temperatures. Creep data for engineering materials are quite sparse due to the expense and time required to develop the experimental data. The machine designer needs to be aware of the creep phenomenon and obtain the latest manufacturer's data on the selected materials if high ambient temperatures are anticipated or if polymers are specified. The creep phenomenon is more complex than this simple description implies. See the bibliography to this chapter for more complete and detailed information on creep in materials.

It is also important to understand that all material properties are a function of temperature, and published test data are usually generated at room temperature. Increased temperature usually reduces strength. Many materials that are ductile at room temperature can behave as brittle materials at low temperatures. Thus, if your application involves either elevated or low temperatures, you need to seek out relevant material-property data for your operating environment. Material manufacturers are the best source of up-to-date information. Most manufacturers of polymers publish creep data for their materials at various temperatures.

2.2 THE STATISTICAL NATURE OF MATERIAL PROPERTIES

Some published data for material properties represent average values of many samples tested. (Other data are stated as minimum values.) The range of variation of the published test data is sometimes stated, sometimes not. Most material properties will vary about the average or mean value according to some statistical distribution such as the *Gaussian* or *normal distribution* shown in Figure 2-11. This curve is defined in terms of two parameters, the *arithmetic mean* μ and the *standard deviation* S_d. The equation of the Gaussian distribution curve is

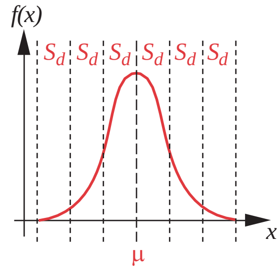


FIGURE 2-11

The Gaussian (Normal) Distribution

$$f(x) = \frac{1}{\sqrt{2\pi} S_d} \exp\left[-\frac{(x-\mu)^2}{2S_d^2}\right], \qquad -\infty \le x \le \infty \tag{2.9a}$$

where x represents some material parameter, $f(x)$ is the frequency with which that value of x occurs in the population, and μ and S_d are defined as

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2.9b}$$

$$S_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2} \tag{2.9c}$$

The *mean* μ defines the most frequently occurring value of x at the peak of the curve, and the *standard deviation* S_d is a measure of the "spread" of the curve about the *mean*. A small value of S_d relative to μ means that the entire population is clustered closely about the mean. A large S_d indicates that the population is widely dispersed about the mean. We can expect to find 68% of the population within $\mu \pm 1S_d$, 95% within $\mu \pm 2S_d$, and 99% within $\mu \pm 3S_d$.

There is considerable scatter in multiple tests of the same material under the same test conditions. Note that there is a 50% chance that the samples of any material that you buy will have a strength less than that material's published mean value. Thus, you may not want to use the mean value alone as a predictor of the strength of a randomly chosen sample of that material. If the standard deviation of the test data is published along with the mean, we can "factor it down" to a lower value that is predictive of some larger percentage of the population based on the ratios listed above. For example, if you want to have a 99% probability that all possible samples of material are stronger than your assumed material strength, you will subtract $3S_d$ from μ to get an allowable value for your design. This assumes that the material property's distribution is Gaussian and not skewed toward one end or the other of the spectrum. If a minimum value of the material property is given (and used), then its statistical distribution is not of concern.

Usually, no data are available on the standard deviation of the material samples tested. But you can still choose to reduce the published mean strength by a reliability factor based on an assumed S_d. One such approach assumes S_d to be some percentage of μ based on experience. Haugen and Wirsching[1] report that the standard deviations of strengths of steels seldom exceed 8% of their mean values. Table 2-2 shows reliability reduction factors based on an assumption of $S_d = 0.08\ \mu$ for various reliabilities. Note that a 50% reliability has a factor of 1 and the factor reduces as you choose higher reli-

Table 2-2
Reliability Factors for $S_d = 0.08\ \mu$

Reliability %	Factor
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620

ability. The reduction factor is multiplied by the mean value of the relevant material property. For example, if you wish 99.99% of your samples to meet or exceed the assumed strength, multiply the mean strength value by 0.702.

In summary, the safest approach is to develop your own material-property data for the particular materials and loading conditions relevant to your design. Since this approach is usually prohibitively expensive in both time and money, the engineer often must rely on published material-property data. Some published strength data are expressed as the minimum strength to be expected in a statistical sample, but other data may be given as the average value for the samples tested. In that case, some of the tested material samples failed at stresses lower than the average value, and your design strength may need to be reduced accordingly.

2.3 HOMOGENEITY AND ISOTROPY

All discussion of material properties so far has assumed that the material is homogeneous and isotropic. **Homogeneous** means that the *material properties are uniform throughout its continuum*, e.g., they are not a function of position. This ideal state is seldom attained in real materials, many of which are subject to the inclusion of discontinuities, precipitates, voids, or bits of foreign matter from their manufacturing process. However, most metals and some nonmetals can be considered, for engineering purposes, to be macroscopically homogeneous despite their microscopic deviations from this ideal.

An **isotropic** material is one whose *mechanical properties are independent of orientation or direction*. That is, the strengths across the width and thickness are the same as along the length of the part, for example. Most metals and some nonmetals can be considered to be macroscopically isotropic. Other materials are **anisotropic**, meaning that *there is no plane of material-property symmetry*. **Orthotropic** materials *have three mutually perpendicular planes of property symmetry and can have different material properties along each axis*. Wood, plywood, fiberglass, and some cold-rolled sheet metals are orthotropic.

One large class of materials that is distinctly nonhomogeneous (i.e., heterogeneous) and nonisotropic is that of **composites** (also see below). Most composites are man-made, but some, such as wood, occur naturally. Wood is a composite of long fibers held together in a resinous matrix of lignin. You know from experience that it is easy to split wood along the grain (fiber) lines and nearly impossible to do so across the grain. Its strength is a function of both orientation and position. The matrix is weaker than the fibers, and it always splits between fibers.

2.4 HARDNESS

The hardness of a material can be an indicator of its resistance to wear (but is not a guarantee of wear resistance). The strengths of some materials such as steels are also closely correlated to their hardness. Various treatments are applied to steels and other metals to increase hardness and strength. These are discussed below.

Hardness is most often measured on one of three scales: *Brinell*, *Rockwell*, or *Vickers*. These hardness tests all involve the forced impression of a small probe into the surface of the material being tested. The **Brinell test** uses a 10-mm hardened steel or tungsten-carbide* ball impressed with either a 500- or 3000-kg load depending on the range of hardness of the material. The diameter of the resulting indent is measured under a microscope and used to calculate the Brinell hardness number, which has the units of kg_f/mm^2. The **Vickers test** uses a diamond-pyramid indenter and measures the width of the indent under the microscope. The **Rockwell test** uses a 1/16-in ball or a 120° cone-shaped diamond indenter and measures the depth of penetration.

Hardness is indicated by a number followed by the letter H, followed by letter(s) to identify the method used, e.g., 375 HB or 396 HV. Several lettered scales (A, B, C, D, F, G, N, T) are used for materials in different Rockwell hardness ranges and it is necessary to specify both the letter and number of a Rockwell reading, such as 60 HRC. In the case of the Rockwell N scale, a narrow-cone-angle diamond indenter is used with loads of 15, 30, or 40 kg and the specification must include the load used as well as the letter specification, e.g., 84.6 HR15N. This Rockwell N scale is typically used to measure the "superficial" hardness of thin or case-hardened materials. The smaller load and narrow-angle N-tip give a shallow penetration that measures the hardness of the case without including effects of the soft core.

All these tests are nondestructive in the sense that the sample remains intact. However, the indentation can present a problem if the surface finish is critical or if the section is thin, so they are actually considered destructive tests. The Vickers test has the advantage of having only one test setup for all materials. Both the Brinell and Rockwell tests require selection of the tip size or indentation load, or both, to match the material tested. The Rockwell test is favored for its lack of operator error, since no microscope reading is required and the indentation tends to be smaller, particularly if the N-tip is used. But, the Brinell hardness number provides a very convenient way to quickly estimate the ultimate tensile strength (S_{ut}) of the material from the relationship

$$\begin{aligned} S_{ut} &\cong 500\,\mathrm{HB} \pm 30\,\mathrm{HB} \quad \text{psi} \\ S_{ut} &\cong 3.45\,\mathrm{HB} \pm 0.2\,\mathrm{HB} \quad \text{MPa} \end{aligned} \tag{2.10}$$

where HB is the Brinell hardness number. This gives a convenient way to obtain a rough experimental measure of the strength of any low- or medium-strength carbon or alloy steel sample, even one that has already been placed in service and cannot be truly destructively tested.

Microhardness tests use a low force on a small diamond indenter and can provide a profile of microhardness as a function of depth across a sectioned sample. The hardness is computed on an absolute scale by dividing the applied force by the area of the indent. The units of **absolute hardness** are kg_f/mm^2. Brinell and Vickers hardness numbers also have these hardness units, though the values measured on the same sample can differ with each method. For example, a Brinell hardness of 500 HB is about the same as a Rockwell C hardness of 52 HRC and an absolute hardness of 600 kg_f/mm^2. Note that these scales are not linearly related, so conversion is difficult. Table 2-3 shows approximate conversions between the Brinell, Vickers, and Rockwell B and C hardness scales for steels and their approximate equivalent ultimate tensile strengths.

* Tungsten carbide is one of the hardest substances known.

2

Table 2-3 Approximate Equivalent Hardness Numbers and Ultimate Tensile Strengths for Steels

Brinell	Vickers	Rockwell		Ultimate, σ_u	
HB	*HV*	*HRB*	*HRC*	MPa	ksi
627	667	—	58.7	2393	347
578	615	—	56.0	2158	313
534	569	—	53.5	1986	288
495	528	—	51.0	1813	263
461	491	—	48.5	1669	242
429	455	—	45.7	1517	220
401	425	—	43.1	1393	202
375	396	—	40.4	1267	184
341	360	—	36.6	1131	164
311	328	—	33.1	1027	149
277	292	—	28.8	924	134
241	253	100	22.8	800	116
217	228	96.4	—	724	105
197	207	92.8	—	655	95
179	188	89.0	—	600	87
159	167	83.9	—	538	78
143	150	78.6	—	490	71
131	137	74.2	—	448	65
116	122	67.6	—	400	58

Note: Load 3000 kg for HB.

Source: Table 5-10, p.185, in N. E. Dowling, *Mechanical Behavior of Materials*, Prentice Hall, Englewood Cliffs, N.J., 1993, with permission.

Heat Treatment

The steel heat-treatment process is quite complicated and is dealt with in detail in materials texts such as those listed in the bibliography at the end of this chapter. The reader is referred to such references for a more complete discussion. Only a brief review of some of the salient points is provided here.

The hardness and other characteristics of many steels and some nonferrous metals can be changed by heat treatment. Steel is an alloy of iron and carbon. The weight percent of carbon present affects the alloy's ability to be heat-treated. A low-carbon steel will have about 0.03 to 0.30% of carbon, a medium-carbon steel about 0.35 to 0.55%, and a high-carbon steel about 0.60 to 1.50%. (Cast irons will have greater than 2% carbon.) Hardenability of steel increases with carbon content. Low-carbon steel has too little carbon for effective through-hardening so other surface-hardening methods must be used (see below). Medium- and high-carbon steels can be through-hardened by appropriate heat treatment. The depth of hardening will vary with alloy content.

QUENCHING To harden a medium- or high-carbon steel, the part is heated above a critical temperature (about 1 400°F {760°C}), allowed to equilibrate for some time, and then suddenly cooled to room temperature by immersion in a water or oil bath. The rapid cooling creates a supersaturated solution of iron and carbon called martensite, which is extremely hard and much stronger than the original soft material. Unfortunately it is also very brittle. In effect, we have traded off the steel's ductility for its increased strength. The rapid cooling also introduces strains to the part. The change in the shape of the stress-strain curve as a result of quenching a ductile, medium-carbon steel is shown in Figure 2-12 (not to scale). While the increased strength is desirable, the severe brittleness of a fully quenched steel usually makes it unusable without tempering.

Tempering Subsequent to quenching, the same part can be reheated to a lower temperature (400–1300°F {200–700°C}), heat-soaked, and then allowed to cool slowly. This will cause some of the martensite to convert to ferrite and cementite, which reduces the strength somewhat but restores some ductility. A great deal of flexibility is possible in terms of tailoring the resulting combination of properties by varying time and temperature during the tempering process. The knowledgeable materials engineer or metallurgist can achieve a wide variety of properties to suit any application. Figure 2-12 also shows a stress-strain curve for the same steel after tempering.

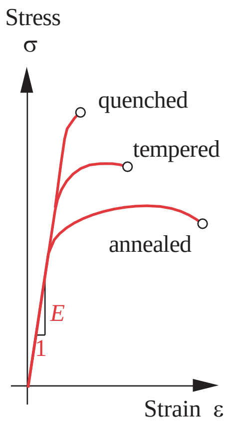


FIGURE 2-12

Stress-Strain Curves for Annealed, Quenched, and Tempered Steel

Annealing The quenching and tempering process is reversible by annealing. The part is heated above the critical temperature (as for quenching) but now allowed to cool slowly to room temperature. This restores the solution conditions and mechanical properties of the unhardened alloy. Annealing is often used even if no hardening has been previously done in order to eliminate any residual stresses and strains introduced by the forces applied in forming the part. It effectively puts the part back into a "relaxed" and soft state, restoring its original stress-strain curve as shown in Figure 2-12.

Normalizing Many tables of commercial steel data indicate that the steel has been normalized after rolling or forming into its stock shape. Normalizing is similar to annealing but involves a shorter soak time at elevated temperature and a more rapid cooling rate. The result is a somewhat stronger and harder steel than a fully annealed one but one that is closer to the annealed condition than to any tempered condition.

Surface (Case) Hardening

When a part is large or thick, it is difficult to obtain uniform hardness within its interior by through hardening. An alternative is to harden only the surface, leaving the core soft. This also avoids the distortion associated with quenching a large, through-heated part. If the steel has sufficient carbon content, its surface can be heated, quenched, and tempered as would be done for through hardening. For low-carbon (mild) steels other techniques are needed to obtain a hardened condition. These involve heating the part in a special atmosphere rich in either carbon, nitrogen or both and then quenching it, a process called *carburizing*, *nitriding*, or *cyaniding*. In all situations, the result is a hard surface (i.e., *case*) on a soft core, referred to as being **case-hardened.**

Carburizing heats low-carbon steel in a carbon monoxide gas atmosphere, causing the surface to take up carbon in solution. **Nitriding** heats low-carbon steel in a nitrogen-gas atmosphere and forms hard iron nitrides in the surface layers. **Cyaniding** heats the part in a cyanide salt bath at about 1 500°F (800°C), and the low-carbon steel takes up both carbides and nitrides from the salt.

For medium- and high-carbon steels no artificial atmosphere is needed, as the steel has sufficient carbon for hardening. Two methods are in common use. **Flame hard-**

ening passes an oxyacetylene flame over the surface to be hardened and follows it with a water jet for quenching. This results in a somewhat deeper hardened case than obtainable from the artificial-atmosphere methods. **Induction hardening** uses electric coils to rapidly heat the part surface, which is then quenched before the core can get hot.

Case hardening by any appropriate method is a very desirable hardening treatment for many applications. It is often advantageous to retain the full ductility (and thus the toughness) of the core material for better energy absorption capacity while also obtaining high hardness on the surface in order to reduce wear and increase surface strength. Large machine parts such as cams and gears are examples of elements that can benefit more from case hardening than from through hardening, as heat distortion is minimized and the tough, ductile core can better absorb impact energy.

Heat Treating Nonferrous Materials

Some nonferrous alloys are hardenable and others are not. Some of the aluminum alloys can be **precipitation hardened**, also called **age hardening**. An example is aluminum alloyed with up to about 4.5% copper. This material can be hot-worked (rolled, forged, etc.) at a particular temperature and then heated and held at a higher temperature to force a random dispersion of the copper in the solid solution. It is then quenched to capture the supersaturated solution at normal temperature. The part is subsequently reheated to a temperature below the quenching temperature and held for an extended period of time while some of the supersaturated solution precipitates out and increases the material's hardness.

Other aluminum alloys, magnesium, titanium, and a few copper alloys are amenable to similar heat treatment. The strengths of the hardened aluminum alloys approach those of medium-carbon steels. Since all aluminum is about 1/3 the density of steel, the stronger aluminum alloys can offer better strength-to-weight ratios than low-carbon (mild) steels.

Mechanical Forming and Hardening

COLD WORKING The mechanical working of metals at room temperature to change their shape or size will also work-harden them and increase their strength at the expense of ductility. Cold working can result from the rolling process in which metal bars are progressively reduced in thickness by being squeezed between rollers, or from any operation that takes the ductile metal beyond the yield point and permanently deforms it. Figure 2-13 shows the process as it affects the material's stress-strain curve. As the load is increased from the origin at *O* beyond the yield point *y* to point *B*, a permanent set *OA* is introduced.

If the load is removed at that point, the stored elastic energy is recovered and the material returns to zero stress at point *A* along a new elastic line *BA* parallel to the original elastic slope *E*. If the load is now reapplied and brought to point *C*, again yielding the material, the new stress-strain curve is *ABCf*. Note that there is now a new yield point *y'* which is at a higher stress than before. The material has **strain-hardened**, increasing its yield strength and reducing its ductility. This process can be repeated until the material becomes brittle and fractures.

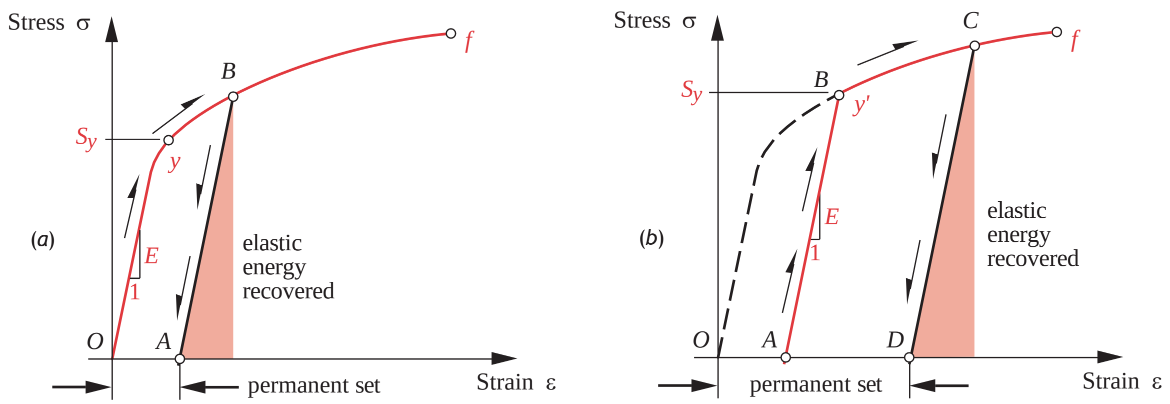


FIGURE 2-13

Strain Hardening a Ductile Material by Cold Working (*a*) First Working (*b*) Second Working

If significant plastic deformation is required for manufacture, such as in making deep-drawn metal pots or cylinders, it is necessary to cold form the material in stages and anneal the part between successive stages to avoid fracture. The annealing resets the material to more nearly the original ductile stress-strain curve, allowing further yielding without fracture.

HOT WORKING All metals have a recrystallization temperature below which the effects of mechanical working will be as described above, e.g., cold worked. If the material is mechanically worked above its recrystallization temperature (hot working), it will tend to at least partially anneal itself as it cools. Thus hot working reduces the strain-hardening problem but introduces its own problems of rapid oxidation of the surface due to the high temperatures. Hot-rolled metals tend to have higher ductility, lower strength and poorer surface finish than cold-worked metals of the same alloy. Hot working does not increase the hardness of the material appreciably, though it can increase the strength by improving grain structure and aligning the "grain" of the metal with the final contours of the part. This is particularly true of forged parts.

FORGING is an automation of the ancient art of blacksmithing. The blacksmith heats the part red-hot in the forge, then beats it into shape with a hammer. When it cools too much for forming, it is reheated and the process is repeated. Forging uses a series of hammer dies shaped to gradually form the hot metal into the final shape. Each stage's die shape represents an achievable change in shape from the original ingot form to the final desired part shape. The part is reheated between blows from the hammer dies which are mounted in a forging press. The large forces required to plastically deform the hot metal require massive presses for parts of medium to large size. Machining operations are required to remove the large "flash" belt at the die parting line and to machine holes, mounting surfaces, etc. The surface finish of a forging is as rough as any hot-rolled part due to oxidation and decarburization of the heated metal.

Virtually any wrought, ductile metal can be forged. Steel, aluminum, and titanium are commonly used. Forging has the advantage of creating stronger parts than casting or machining can. Casting alloys are inherently weaker in tension than wrought alloys. The hot forming of a wrought material into the final forged shape causes the material's

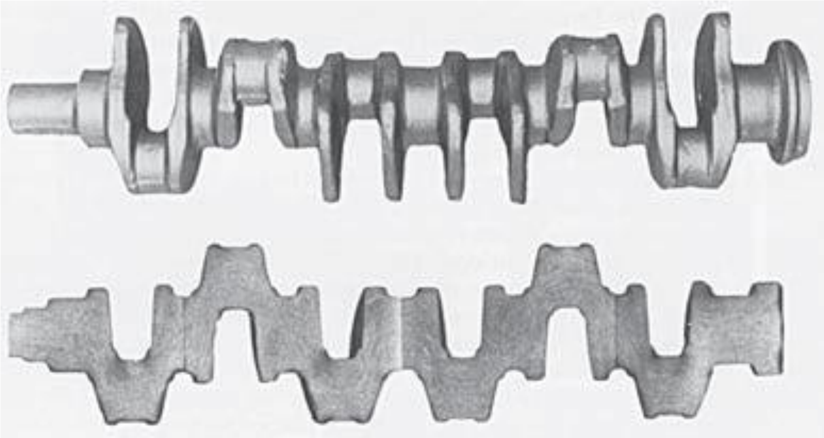


FIGURE 2-14

Forged Steel Crankshaft for a Diesel-Truck Engine - *Courtesy of Wyman-Gordon Corp., Grafton, MA*

internal "flow lines" or "grain" to approximate the contours of the part, which can result in greater strength than if a stock shape's flow lines were severed by machining to the final contour. Forgings are used in highly stressed parts, such as aircraft wing and fuselage structures, engine crankshafts and connecting rods, and vehicle suspension links. Figure 2-14 shows a forged truck crankshaft. In the cross section, the grain lines can be seen to follow the crankshaft's contours. The high cost of the multiple dies needed for forged shapes makes it an impractical choice unless production quantities are large enough to amortize the tooling cost.

EXTRUSION is used principally for nonferrous metals (especially aluminum) as it typically uses steel dies. The usual die is a thick, hardened-tool-steel disk with a tapered "hole" or orifice ending in the cross-sectional shape of the finished part. A billet of the extrudate is heated to a soft state and then rammed at fairly high speed through the die, which is clamped in the machine. The billet flows, or extrudes, into the die's shape. The process is similar to the making of macaroni. A long strand of the material in the desired cross section is extruded from the billet. The extrusion then passes through a water-spray cooling station. Extrusion is an economical way to obtain custom shapes of constant cross section since the dies are not very expensive to make. Dimensional control and surface finish are good. Extrusion is used to make aluminum mill shapes such as angles, channels, I-beams, and custom shapes for storm-door and -window frames, sliding-door frames, etc. The extrusions are cut and machined as necessary to assemble them into the finished product. Some extruded shapes are shown in Figure 2-15.

FIGURE 2-15

Extrusions - *Courtesy of The Aluminum Extruders Council*

2.5 COATINGS AND SURFACE TREATMENTS

Many coatings and surface treatments are available for metals. Some have the prime purpose of inhibiting corrosion while others are intended to improve surface hardness and wear resistance. Coatings are also used to change dimensions (slightly) and to alter physical properties such as reflectance, color, and resistivity. For example, piston

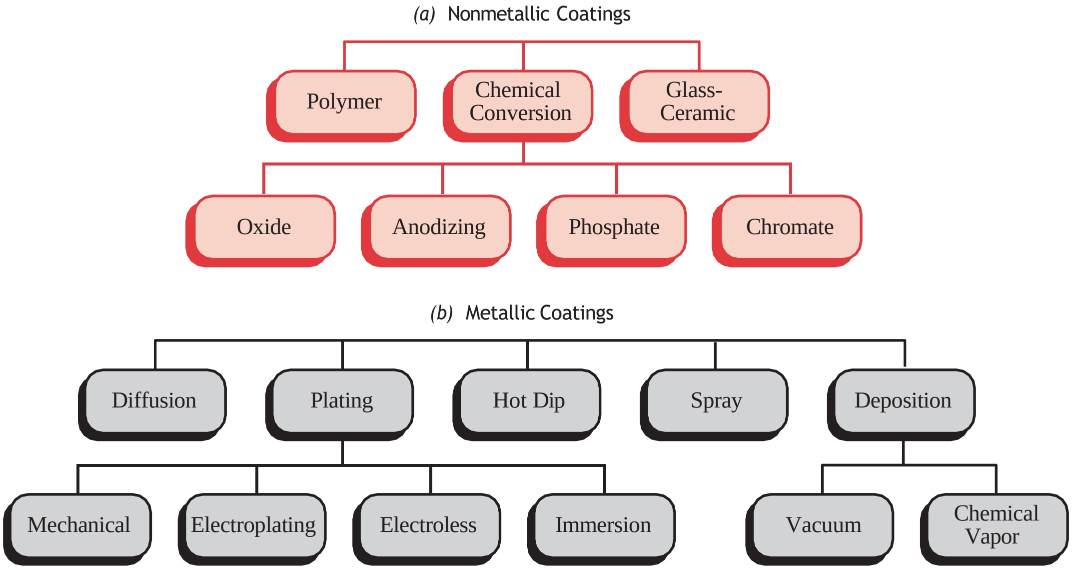


FIGURE 2-16

Coating Methods Available for Metals

rings are chrome-plated to improve wear resistance, fasteners are plated to reduce corrosion, and automobile trim is chrome-plated for appearance and corrosion resistance. Figure 2-16 shows a chart of various types of coatings for machine applications. These divide into two major classes, metallic and nonmetallic, based on the type of coating, not substrate. Some of the classes divide into many subclasses. We will discuss only a few of these here. The reader is encouraged to seek more information from the references in the bibliography.

Galvanic Action

When a coating of one metal is applied to another dissimilar metal, a galvanic cell may be created. All metals are electrolytically active to a greater or lesser degree and if sufficiently different in their electrolytic potential will create a battery in the presence of a conductive electrolyte such as seawater or even tap water. Table 2-4 lists some common metals ordered in terms of their galvanic action potential from the least noble (most electrolytically active) to the most noble (least active). Combinations of metals that are close to each other in the galvanic series, such as cast iron and steel, are relatively safe from galvanic corrosion. Combinations of metals far apart on this scale, such as aluminum and copper, will experience severe corrosion in an electrolyte or even in a moist environment.

In a conductive medium, the two metals become anode and cathode, with the less-noble metal acting as the anode. The self-generated electrical current flow causes a loss of material from the anode and a deposition of material on the cathode. The less-noble metal gradually disappears. This problem occurs whenever two metals sufficiently far apart in the galvanic series are present in an electrically conductive medium. Thus, not only coatings but fasteners and mating parts must be made of metal combinations that will not create this problem.

Table 2-4
Galvanic Series of Metals in Seawater

Least noble
Magnesium
Zinc
Aluminum
Cadmium
Steel
Cast iron
Stainless steel
Lead
Tin
Nickel
Brass
Copper
Bronze
Monel
Silver
Titanium
Graphite
Gold
Platinum
Most noble

Electroplating

Electroplating involves the deliberate creation of a galvanic cell in which the part to be plated is the cathode and the plating material is the anode. The two metals are placed in an electrolyte bath and a direct current applied from anode to cathode. Ions of the plating material are driven to the plating substrate through the electrolyte and cover the part with a thin coating of the plating material. Allowance must be made for the plating thickness, which is controllable. Plating thickness is uniform except at sharp corners or in holes and crevices. The plating builds up on the outside corners and will not go into holes or narrow crevices. Thus, grinding may be necessary after plating to restore dimensions. Worn parts (or mistakes) can sometimes be repaired by plating on a coating of suitable material, then regrinding to dimension.

Steels, nickel- and copper-based alloys, as well as other metals are readily electroplatable. Two approaches are possible. If a more noble (less active) metal is plated onto the substrate, it can reduce the tendency to oxidize as long as the plating remains intact to protect the substrate from the environment. Tin, nickel, and chromium are often used to electroplate steel for corrosion resistance. Chrome plating also offers an increase in surface hardness to HRC 70, which is above that obtainable from many hardened alloy steels.* Unfortunately, any disruptions or pits in the plating can provide nodes for galvanic action if conductive media (such as rainwater) are present. Because the substrate is less noble than the plating, it becomes the sacrificial anode and rapidly corrodes. Electroplating with metals more noble than the substrate is seldom used for parts that will be immersed in water or other electrolytes.

Alternatively, a less-noble metal can be plated onto the substrate to serve as a sacrificial anode which will corrode instead of the substrate. The most common example of this is zinc coating of steel, also called galvanizing. (Cadmium can be used instead of zinc and will last longer in saltwater or salt-air environments.) The zinc or cadmium coating will gradually corrode and protect the more noble steel substrate until the coating is used up, after which the steel will oxidize. Zinc coating can be applied by a process called “hot dipping” rather than by electroplating, which will result in a thicker and more protective coating recognizable by its “mother-of-pearl” appearance. Galvanizing is often applied by manufacturers to automobile body panels to inhibit corrosion. Sacrificial zinc anodes are also attached to aluminum outboard motors and aluminum boat hulls to short-circuit corrosion of the aluminum in seawater.

A caution about electroplated coatings is that hydrogen embrittlement of the substrate can occur, causing significant loss of strength. Electroplated finishes should not be used on parts that are fatigue loaded. Experience has shown that electroplating severely reduces the fatigue strength of metals and can cause early failure.

Electroless Plating

Electroless plating puts a coating of nickel on the substrate without any electric current needed. The substrate “cathode” in this case (there is no anode) acts as a catalyst to start a chemical reaction that causes nickel ions in the electrolyte solution to be reduced and deposited on the substrate. The nickel coating also acts as a catalyst and keeps the reaction going until the part is removed from the bath. Thus, relatively thick coatings can be developed. Coatings are typically between 0.001 in and 0.002 in thick. Unlike elec-

* It is interesting to note that chromium in the pure form is softer than hardened steel but when electroplated onto steel, it becomes harder than the steel substrate. Nickel and iron also increase their hardness when electroplated on metal substrates. The mechanism is not well understood, but it is believed that internal microstrains are developed in the plating process that harden the coating. The hardness of the plating can be controlled by changes in process conditions.

troplating, the electroless nickel plate is completely uniform and will enter holes and crevices. The plate is dense and fairly hard at around 43 HRC. Other metals can also be electroless plated but nickel is most commonly used.

Anodizing

While aluminum can be electroplated (with difficulty), it is more common to treat it by anodizing. This process creates a very thin layer of aluminum oxide on the surface. The aluminum oxide coating is self-limiting in that it prevents atmospheric oxygen from further attacking the aluminum substrate in service. The anodized oxide coating is naturally colorless but dyes can be added to color the surface and provide a pleasing appearance in a variety of hues. This is a relatively inexpensive surface treatment with good corrosion resistance and negligible distortion. Titanium, magnesium, and zinc can also be anodized.

A variation on conventional anodizing of aluminum is so-called "hard anodizing." Since aluminum oxide is a ceramic material, it is naturally very hard and abrasion resistant. **Hard anodizing** provides a thicker (but not actually harder) coating than conventional anodizing and is often used to protect the relatively soft aluminum parts from wear in abrasive contact situations. The hardness of this surface treatment exceeds that of the hardest steel, and hard-anodized aluminum parts can be run against hardened steel though the somewhat abrasive aluminum oxide surface is not kind to the steel.

Plasma-Sprayed Coatings

A variety of very hard ceramic coatings can be applied to steel and other metal parts by a plasma-spray technique. The application temperatures are high, which limits the choice of substrate. The coatings as sprayed have a rough "orange-peel" surface finish which requires grinding or polishing to obtain a fine finish. The main advantage is a surface with extremely high hardness and chemical resistance. However, the ceramic coatings are brittle and subject to chipping under mechanical or thermal shock.

Chemical Coatings

The most common chemical treatments for metals range from a phosphoric acid wash on steel (or chromatic acid on aluminum) that provides limited and short-term oxidation resistance, to paints of various types designed to give more lasting corrosion protection. Paints are available in a large variety of formulations for different environments and substrates. One-part paints give somewhat less protection than two-part epoxy formulations, but all chemical coatings should be viewed as only temporary protection against corrosion, especially when used on corrosion-prone materials such as steel. Baked enamel and porcelain finishes on steel have longer lives in terms of corrosion resistance, though they suffer from brittleness. New formulations of paints and protective coatings are continually being developed. The latest and best information will be obtained from vendors of these products.

2.6 GENERAL PROPERTIES OF METALS

The large variety of useful engineering materials can be confusing to the beginning engineer. There is not space enough in this book to deal with the topic of material selection in complete detail. Several references are provided in this chapter's bibliography which the reader is encouraged to use. Tables of mechanical property data are also provided for a limited set of materials in Appendix A of this book. Figure 2-17 shows the Young's moduli for several engineering metals.

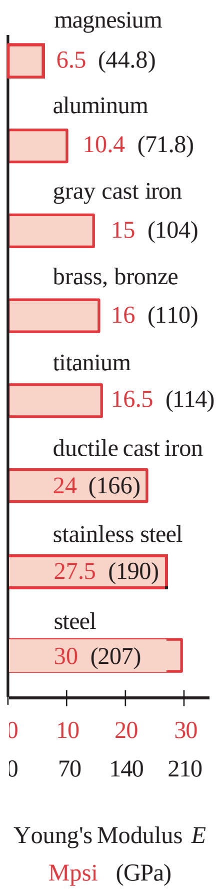


FIGURE 2-17

Young's Moduli for Various Metals

The following sections attempt to provide some general information and guidelines for the engineer to help identify what types of materials might be suitable in a given design situation. It is expected that the practicing engineer will rely heavily on the expertise and help available from materials manufacturers in selecting the optimum material for each design. Many references are also published which list detailed property data for most engineering materials. Some of these references are listed in the bibliography to this chapter.

Cast Iron

Cast irons constitute a whole family of materials. Their main advantages are relatively low cost and ease of fabrication. Some are weak in tension compared to steels but, like most cast materials, have high compressive strengths. Their densities are slightly lower than steel at about 0.25 lb/in^3 (6 920 kg/m^3). Most cast irons do not exhibit a linear stress-strain relationship below the elastic limit; they do not obey Hooke's law. Their modulus of elasticity E is estimated by drawing a line from the origin through a point on the curve at 1/4 the ultimate tensile strength and is in the range of 14–25 Mpsi (97–172 MPa). Cast iron's chemical composition differs from steel principally in its higher carbon content, being between 2 and 4.5%. The large amount of carbon, present in some cast irons as graphite, makes some of these alloys easy to pour as a casting liquid and also easy to machine as a solid. The most common means of fabrication is sand casting with subsequent machining operations. Cast irons are not easily welded, however.

White Cast Iron is a very hard and brittle material. It is difficult to machine and has limited uses, such as in linings for cement mixers where its hardness is needed.

Gray Cast Iron is the most commonly used form of cast iron. Its graphite flakes give it its gray appearance and name. The ASTM grades gray cast iron into seven classes based on the minimum tensile strength in kpsi. Class 20 has a minimum tensile strength of 20 kpsi (138 MPa). The class numbers of 20, 25, 30, 35, 40, 50, and 60 then represent the tensile strength in kpsi. Cost increases with increasing tensile strength. This alloy is easy to pour, easy to machine, and offers good acoustical damping. This makes it the popular choice for machine frames, engine blocks, brake rotors and drums, etc. The graphite flakes also give it good lubricity and wear resistance. Its relatively low tensile strength recommends against its use in situations where large bending or fatigue loads are present, though it is sometimes used in low-cost engine crankshafts. It runs reasonably well against steel if lubricated.

Malleable Cast Iron has superior tensile strength to gray cast iron but does not wear as well. The tensile strength can range from 50 to 120 kpsi (345 to 827 MPa) depending on formulation. It is often used in parts where bending stresses are present.

Nodular (Ductile) Cast Iron has the highest tensile strength of the cast irons, ranging from about 70 to 135 kpsi (480 to 930 MPa). The name *nodular* comes from the fact that its graphite particles are spheroidal in shape. Ductile cast iron has a higher modulus of elasticity (about 25 Mpsi {172 GPa}) than gray cast iron and exhibits a linear stress-strain curve. It is tougher, stronger, more ductile, and less porous than gray cast iron. It is the cast iron of choice for fatigue-loaded parts such as crankshafts, pistons, and cams.

Cast Steels

Cast steel is similar to wrought steel in terms of its chemical content, i.e., it has much less carbon than cast iron. The mechanical properties of cast steel are superior to cast iron but inferior to wrought steel. Its principal advantage is ease of fabrication by sand or investment (lost wax) casting. Cast steel is classed according to its carbon content into low carbon (< 0.2%), medium carbon (0.2–0.5%) and high carbon (> 0.5%). Alloy cast steels are also made containing other elements for high strength and heat resistance. The tensile strengths of cast steel alloys range from about 65 to 200 kpsi (450 to 1380 MPa).

Wrought Steels

The term "wrought" refers to all processes that manipulate the shape of the material without melting it. Hot rolling and cold rolling are the two most common methods used though many variants exist, such as wire drawing, deep drawing, extrusion, and cold-heading. The common denominator is a deliberate yielding of the material to change its shape either at room or at elevated temperatures.

Hot-Rolled Steel is produced by forcing hot billets of steel through sets of rollers or dies which progressively change their shape into I-beams, channel sections, angle irons, flats, squares, rounds, tubes, sheets, plates, etc. The surface finish of hot-rolled shapes is rough due to oxidation at the elevated temperatures. The mechanical properties are also relatively low because the material ends up in an annealed or normalized state unless deliberately heat-treated later. This is the typical choice for low-carbon structural steel members used for building- and machine-frame construction. Hot-rolled material is also used for machine parts that will be subjected to extensive machining (gears, cams, etc.) where the initial finish of the stock is irrelevant and uniform, non-cold-worked material properties are desired in advance of a planned heat treatment. A wide variety of alloys and carbon contents are available in hot-rolled form.

Cold-Rolled Steel is produced from billets or hot-rolled shapes. The shape is brought to final form and size by rolling between hardened steel rollers or drawing through dies at room temperature. The rolls or dies burnish the surface and cold work the material, increasing its strength and reducing its ductility as was described in the section on mechanical forming and hardening above. The result is a material with good surface finish and accurate dimensions compared to hot-rolled material. Its strength and hardness are increased at the expense of significant built-in strains, which can later be released during machining, welding, or heat treating, then causing distortion. Cold-rolled shapes commonly available are sheets, strips, plates, round and rectangular bars, tubes, etc. Structural shapes such as I-beams are typically available only as hot rolled.

Steel Numbering Systems

Several steel numbering systems are in general use. The ASTM, AISI, and SAE* have devised codes to define the alloying elements and carbon content of steels. Table 2-5 lists some of the AISI/SAE designations for commonly used steel alloys. The first two digits indicate the principal alloying elements. The last two digits indicate the amount of carbon present, expressed in hundredths of a percent. ASTM and the SAE have developed a new Unified Numbering System for all metal alloys, which uses the prefix UNS followed by a letter and a 5-digit number. The letter defines the alloy category, *F* for cast iron, *G* for carbon and low-alloy steels, *K* for special-purpose steels, *S* for stainless steels, and *T* for tool steels. For the *G* series, the numbers are the same as the AISI/SAE designations in Table 2-5 with a trailing zero added. For example, SAE 4340 becomes UNS G43400. See reference 2 for more information on metal numbering systems. We will use the AISI/SAE designations for steels.

* ASTM is the American Society for Testing and Materials, AISI is the American Iron and Steel Institute, and SAE is the Society of Automotive Engineers. AISI and SAE both use the same designations for steels.

Plain Carbon Steel is designated by a first digit of 1 and a second digit of 0, since no alloys other than carbon are present. The low-carbon steels are those numbered AISI 1005 to 1030, medium-carbon from 1035 to 1055, and high-carbon from 1060 to 1095. The AISI 11xx series adds sulphur, principally to improve machinability. These are called free-machining steels and are not considered alloy steels as the sulphur does

Table 2-5 AISI/SAE Designations of Steel Alloys

A partial list - other alloys are available - consult the manufacturers

Type	AISI/SAE Series	Principal Alloying Elements
Carbon Steels		
Plain	10xx	Carbon
Free-cutting	11xx	Carbon plus Sulphur (resulphurized)
Alloy Steels		
Manganese	13xx	1.75% Manganese
	15xx	1.00 to 1.65% Manganese
Nickel	23xx	3.50% Nickel
	25xx	5.00% Nickel
Nickel-Chrome	31xx	1.25% Nickel and 0.65 or 0.80% Chromium
	33xx	3.50% Nickel and 1.55% Chromium
Molybdenum	40xx	0.25% Molybdenum
	44xx	0.40 or 0.52% Molybdenum
Chrome-Moly	41xx	0.95% Chromium and 0.20% Molybdenum
Nickel-Chrome-Moly	43xx	1.82% Nickel, 0.50 or 0.80% Chromium, and 0.25% Molybdenum
	47xx	1.45% Nickel, 0.45% Chromium, and 0.20 or 0.35% Molybdenum
Nickel-Moly	46xx	0.82 or 1.82% Nickel and 0.25% Molybdenum
	48xx	3.50% Nickel and 0.25% Molybdenum
Chrome	50xx	0.27 to 0.65% Chromium
	51xx	0.80 to 1.05% Chromium
	52xx	1.45% Chromium
Chrome-Vanadium	61xx	0.60 to 0.95% Chromium and 0.10 to 0.15% Vanadium minimum

not improve the mechanical properties and also makes it brittle. The ultimate tensile strength of plain carbon steel can vary from about 60 to 150 kpsi (414 to 1 034 MPa) depending on heat treatment.

ALLOY STEELS have various elements added in small quantities to improve the material's strength, hardenability, temperature resistance, corrosion resistance, and other properties. Any level of carbon can be combined with these alloying elements. Chromium is added to improve strength, ductility, toughness, wear resistance, and hardenability. Nickel is added to improve strength without loss of ductility, and it also enhances case hardenability. Molybdenum, used in combination with nickel and/or chromium, adds hardness, reduces brittleness, and increases toughness. Many other alloys in various combinations, as shown in Table 2-5, are used to achieve specific properties. Specialty steel manufacturers are the best source of information and assistance for the engineer trying to find the best material for any application. The ultimate tensile strength of alloy steels can vary from about 80 to 300 kpsi (550 to 2 070 MPa), depending on its alloying elements and heat treatment. Appendix A contains tables of mechanical property data for a selection of carbon and alloy steels. Figure 2-18 shows approximate ultimate tensile strengths of some normalized carbon and alloy steels and Figure 2-19 shows engineering stress-strain curves from tensile tests of three steels.

TOOL STEELS are medium- to high-carbon alloy steels especially formulated to give very high hardness in combination with wear resistance and sufficient toughness to resist the shock loads experienced in service as cutting tools, dies and molds. There is a very large variety of tool steels available. Refer to the bibliography and to manufacturers' literature for more information.

STAINLESS STEELS are alloy steels containing at least 10% chromium and offer much improved corrosion resistance over plain or alloy steels, though their name should not be taken too literally. Stainless steels will stain and corrode (slowly) in severe environments such as seawater. Some stainless-steel alloys have improved resistance to high

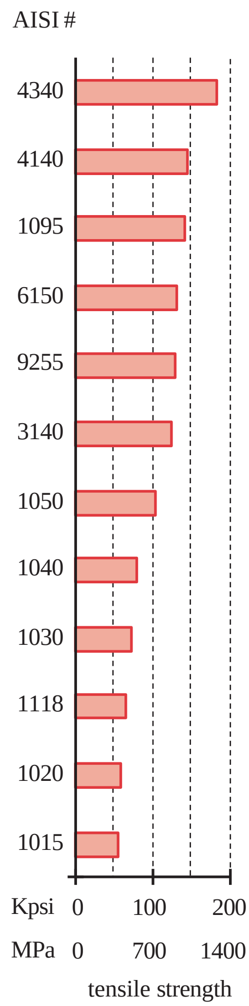


FIGURE 2-18

Approximate Ultimate Tensile Strengths of Some Normalized Steels

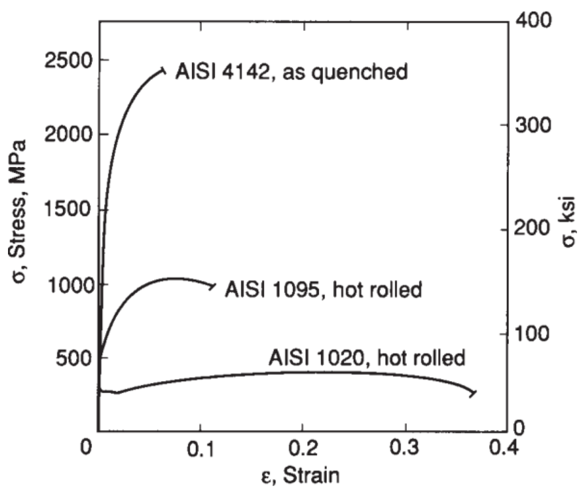


FIGURE 2-19

Tensile Test Stress-Strain Curves of Three Steel Alloys (From Fig. 5.16, p. 160, in N. E. Dowling, *Mechanical Behavior of Materials*, Prentice-Hall, Englewood Cliffs, N.J., 1993, with permission) temperature. There are four types of stainless steel, called **martensitic, ferritic, austenitic**, and **precipitation hardening**.

Martensitic stainless steel contains 11.5 to 15% Cr and 0.15 to 1.2% C, is magnetic, can be hardened by heat treatment, and is commonly used for cutlery. **Ferritic** stainless steel has over 16% Cr and a low carbon content, is magnetic, soft, and ductile, but is not heat treatable though its strength can be increased modestly by cold working. It is used for deep-drawn parts such as cookware and has better corrosion resistance than the martensitic SS. The ferritic and martensitic stainless steels are both called **400 series** stainless steel.

Austenitic stainless steel is alloyed with 17 to 25% chromium and 10 to 20% nickel. It has better corrosion resistance due to the nickel, is nonmagnetic, and has excellent ductility and toughness. It cannot be hardened except by cold working. It is classed as **300 series** stainless steel.

Precipitation-hardening stainless steels are designated by their alloy percentages followed by the letters PH, as in 17-4 PH which contains 17% chromium and 4% nickel. These alloys offer high strength and high temperature and corrosion resistance.

The **300 series** stainless steels are very weldable but the 400 series are less so. All grades of stainless steel have poorer heat conductivity than regular steel and many of the stainless alloys are difficult to machine. All stainless steels are significantly more expensive than regular steel. See Appendix A for mechanical property data.

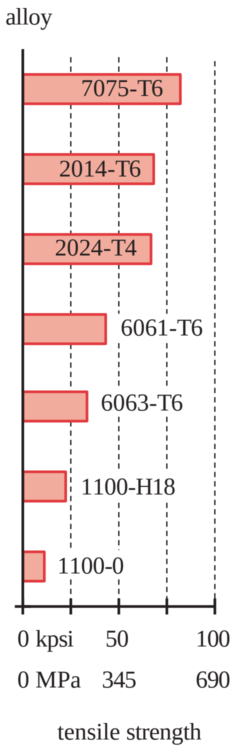


FIGURE 2-20

Ultimate Tensile Strengths of Some Aluminum Alloys

Aluminum

Aluminum is the most widely used nonferrous metal, being second only to steel in world consumption. Aluminum is produced in both "pure" and alloyed forms. Aluminum is commercially available up to 99.8% pure. The most common alloying elements are copper, silicon, magnesium, manganese, and zinc, in varying amounts up to about 5%. The principal advantages of aluminum are its low density, good strength-to-weight ratio (SWR), ductility, excellent workability, castability, and weldability, corrosion resistance, high conductivity, and reasonable cost. Compared to steel it is 1/3 as dense (0.10 lb/in^3 versus 0.28 lb/in^3), about 1/3 as stiff (E = 10.3 Mpsi {71 GPa} versus 30 Mpsi {207 GPa}), and generally less strong. If you compare the strengths of low-carbon steel and pure aluminum, the steel is about three times as strong. Thus the specific strength is approximately the same in that comparison. However, pure aluminum is seldom used in engineering applications. It is too soft and weak. Pure aluminum's principal advantages are its bright finish and good corrosion resistance. It is used mainly in decorative applications.

The aluminum alloys have significantly greater strengths than pure aluminum and are used extensively in engineering, with the aircraft and automotive industries among the largest users. The higher-strength aluminum alloys have tensile strengths in the 70 to 90 kpsi (480 to 620 MPa) range, and yield strengths about twice that of mild steel. They compare favorably to medium-carbon steels in specific strength. Aluminum competes successfully with steel in some applications, though few materials can beat steel if very high strength is needed. See Figure 2-20 for tensile strengths of some aluminum alloys. Figure 2-21 shows tensile-test engineering stress-strain curves for three aluminum alloys. Aluminum's strength is reduced at low temperatures as well as at elevated temperatures.

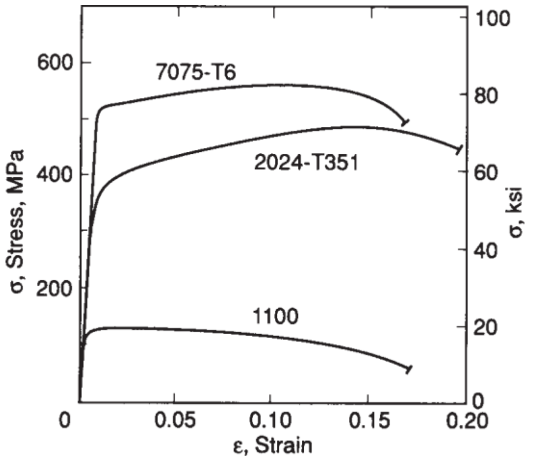


FIGURE 2-21

Tensile Test Stress-Strain Curves of Three Aluminum Alloys (From Fig. 5.17, p. 160, in N. E. Dowling, *Mechanical Behavior of Materials*, Prentice-Hall, Englewood Cliffs, N.J., 1993, with permission)

Some aluminum alloys are hardenable by heat treatment and others by strain hardening or precipitation and aging. High-strength aluminum alloys are about 1.5 times harder than soft steel, and surface treatments such as *hard anodizing* can bring the surface to a condition harder than the hardest steel.

Aluminum is among the most easily worked of the engineering materials, though it tends to work harden. It casts, machines, welds,* and hot and cold forms† easily. It can also be extruded. Alloys are specially formulated for both sand and die casting as well as for wrought and extruded shapes and for forged parts.

Wrought-Aluminum Alloys are available in a wide variety of stock shapes such as I-beams, angles, channels, bars, strip, sheet, rounds, and tubes. Extrusion allows relatively inexpensive custom shapes as well. The Aluminum Association numbering system for alloys is shown in Table 2-6. The first digit indicates the principal alloying element and defines the series. Hardness is indicated by a suffix containing a letter and up to 3 numbers as defined in the table. The most commonly available and most-used aluminum alloys in machine-design applications are the 2000 and 6000 series.

The oldest aluminum alloy is 2024, which contains 4.5% copper, 1.5% magnesium, and 0.8% manganese. It is among the most machinable of the aluminum alloys and is heat treatable. In the higher tempers, such as -T3 and -T4, it has a tensile strength approaching 70 kpsi (483 MPa), which also makes it one of the strongest of the aluminum alloys. It also has high fatigue strength. However, it has poor weldability and formability compared to the other aluminum alloys.

The 6061 alloy contains 0.6% silicon, 0.27% copper, 1.0% manganese, and 0.2% chromium. It is widely used in structural applications because of its excellent weldability. Its strength is about 40 to 45 kpsi (276 to 310 MPa) in the higher tempers. It has lower fatigue strength than 2024 aluminum. It is easily machined and is a popular alloy for extrusion, which is a hot-forming process.

The 7000 series is called aircraft aluminum and is used mostly in airframes. These are the strongest alloys of aluminum with tensile strengths up to 98 kpsi (676 MPa) and

* The heat of welding causes localized annealing, which can remove the desirable strengthening effects of cold work or heat treatment in any metal.

† Some aluminum alloys will cold-work when formed to the degree that trying to bend them again (without first annealing) will cause fractures. Some bicycle racers prefer steel frames over aluminum despite their added weight because, once an aluminum frame is bent in a fall, it cannot be straightened without cracking. Damaged steel tube frames can be straightened and reused.

Table 2-6 Aluminum Association Designations of Aluminum Alloys
A partial list - other alloys are available - consult the manufacturers

Series	Major Alloying Elements	Secondary Alloys
1xxx	Commercially pure (99%)	None
2xxx	Copper (Cu)	Mg, Mn, Si
3xxx	Manganese (Mn)	Mg, Cu
4xxx	Silicon (Si)	None
5xxx	Magnesium (Mg)	Mn, Cr
6xxx	Magnesium and Silicon	Cu, Mn
7xxx	Zinc (Zn)	Mg, Cu, Cr
Hardness Designations		
xxxx-F	As fabricated	
xxxx-O	Annealed	
xxxx-Hyyy	Work hardened	
xxxx-Tyyy	Thermal/age hardened	

the highest fatigue strength of about 22 kpsi (152 MPa) @ 10^8 cycles. Some alloys are also available in an *alclad* form which bonds a thin layer of pure aluminum to one or both sides to improve corrosion resistance.

Cast-Aluminum Alloys are differently formulated than the wrought alloys. Some of these are hardenable but their strength and ductility are less than those of the wrought alloys. Alloys are available for sand casting, die casting, or investment casting. See Appendix A for mechanical properties of wrought- and cast-aluminum alloys.

Titanium

Though discovered as an element in 1791, commercially produced titanium has been available only since the 1940s, so it is among the newest of engineering metals. Titanium can be the answer to an engineer's prayer in some cases. It has an upper service-temperature limit of 1 200 to 1 400°F (650 to 750°C), weighs half as much as steel (0.16 lb/in^3 {4 429 kg/m^3}), and is as strong as a medium-strength steel (135 kpsi {930 MPa} typical). Its Young's modulus is 16 to 18 Mpsi (110 to 124 GPa), or about 60% that of steel. Its specific strength approaches that of the strongest alloy steels and exceeds that of medium-strength steels by a factor of 2. Its specific stiffness is greater than that of steel, making it as good or better in limiting deflections. It is also nonmagnetic.

Titanium is very corrosion resistant and is nontoxic, allowing its use in contact with acidic or alkaline foodstuffs and chemicals, and in the human body as replacement heart valves and hip joints, for example. Unfortunately, it is expensive compared to aluminum and steel. It finds much use in the aerospace industry, especially in military aircraft structures and in jet engines, where strength, light weight, and high temperature and corrosion resistance are all required.

Titanium is available both pure and alloyed with combinations of aluminum, vanadium, silicon, iron, chromium, and manganese. Its alloys can be hardened and anodized. Limited stock shapes are available commercially. It can be forged and wrought, though it is quite difficult to cast, machine, and cold form. Like steel and unlike most other metals, some titanium alloys exhibit a true endurance limit, or leveling off of the fatigue strength, beyond about 10^6 cycles of repeated loading, as shown in Figure 2-10. See Appendix A for mechanical property data.

Magnesium

Magnesium is the lightest of commercial metals but is relatively weak. The tensile strengths of its alloys are between 10 and 50 kpsi (69 and 345 MPa). The most common alloying elements are aluminum, manganese, and zinc. Because of its low density (0.065 lb/in^3 {1 800 kg/m^3}), its specific strength approaches that of aluminum. Its Young's modulus is 6.5 Mpsi (45 GPa) and its specific stiffness exceeds those of aluminum and steel. It is very easy to cast and machine but is more brittle than aluminum and thus is difficult to cold form.

It is nonmagnetic and has fair corrosion resistance, better than steel, but not as good as aluminum. Some magnesium alloys are hardenable, and all can be anodized. It is the most active metal on the galvanic scale and cannot be combined with most other metals in a wet environment. It is also extremely flammable, especially in powder or chip form, and its flame cannot be doused with water. Machining requires flooding with oil coolant to prevent fire. It is roughly twice as costly per pound as aluminum. Magnesium is used where light weight is of paramount importance such as in castings for chain-saw housings and other hand-held items. See Appendix A for mechanical property data.

Copper Alloys

Pure copper is soft, weak, and malleable and is used primarily for piping, flashing, electrical conductors (wire) and motors. It cold works readily and can become brittle after forming, requiring annealing between successive draws.

Many alloys are possible with copper. The most common are brasses and bronzes which themselves are families of alloys. **Brasses**, in general, are alloys of copper and zinc in varying proportions and are used in many applications, from artillery shells and bullet shells to lamps and jewelry.

Bronzes were originally defined as alloys of copper and tin, but now also include alloys containing no tin, such as silicon bronze and aluminum bronze, so the terminology is somewhat confusing. **Silicon bronze** is used in marine applications such as ship propellers.

Beryllium copper is neither brass nor bronze and is the strongest of the alloys, with strengths approaching those of alloy steels (200 kpsi {1 380 MPa}). It is often used in springs that must be nonmagnetic, carry electricity, or exist in corrosive environments. **Phosphor bronze** is also used for springs but unlike beryllium copper, it cannot be bent along the grain or heat treated.

Copper and its alloys have excellent corrosion resistance and are nonmagnetic. All copper alloys can be cast, hot or cold formed, and machined, but pure copper is difficult to machine. Some alloys are heat treatable and all will work harden. The Young's modulus of most copper alloys is about 17 Mpsi (117 GPa) and their weight density is slightly higher than that of steel at 0.31 lb/in^3 (8 580 kg/m^3). Copper alloys are expensive compared to other structural metals. See Appendix A for mechanical property data.

2.7 GENERAL PROPERTIES OF NONMETALS

The use of nonmetallic materials has increased greatly in the last 50 years. The usual advantages sought are light weight, corrosion resistance, temperature resistance, dielectric strength, and ease of manufacture. Cost can range from low to high compared to metals depending on the particular nonmetallic material. There are three general categories of nonmetals of general engineering interest: **polymers** (plastics), **ceramics,** and **composites**.

Polymers have a wide variety of properties, principally low weight, relatively low strength and stiffness, good corrosion and electrical resistance, and relatively low cost per unit volume. **Ceramics** can have extremely high compressive (but not tensile) strengths, high stiffness, high temperature resistance, high dielectric strength (resistance to electrical current), high hardness, and relatively low cost per unit volume. **Composites** can have almost any combination of properties you want to build into them, including the highest specific strengths obtainable from any materials. Composites can be low or very high in cost. We briefly discuss nonmetals and some of their applications. Space does not permit a complete treatment of these important classes of materials. The reader is directed to the bibliography for further information. Appendix A also provides some mechanical property data for polymers.

Polymers

The word polymers comes from **poly** = *many* and **mers** = *molecules*. Polymers are long-chain molecules of organic materials or carbon-based compounds. (There is also a family of silicon-based polymeric compounds.) The source of most polymers is oil or coal, which contains the carbon or hydrocarbons necessary to create the polymers. While there are many natural polymer compounds (wax, rubber, proteins, ...), most polymers used in engineering applications are man-made. Their properties can be tailored over a wide range by copolymerization with other compounds or by alloying two or more polymers together. Mixtures of polymers and inorganic materials such as talc or glass fiber are also common.

Because of their variety, it is difficult to generalize about the mechanical properties of polymers, but compared to metals they have low density, low strength, low stiffness, nonlinear elastic stress-strain curves as shown in Figure 2-22 (with a few exceptions), low hardness, excellent electrical and corrosion resistance, and ease of fabrication. Their apparent moduli of elasticity vary widely from about 10 kpsi (69 MPa) to about 400 kpsi (2.8 GPa), all much less stiff than any metals. Their ultimate tensile strengths range from about 4 kpsi (28 MPa) for the weakest unfilled polymer to about 22 kpsi (152 MPa) for the strongest glass-filled polymers. The specific gravities of most polymers range from about 0.95 to 1.8 compared to about 2 for magnesium, 3 for alu-

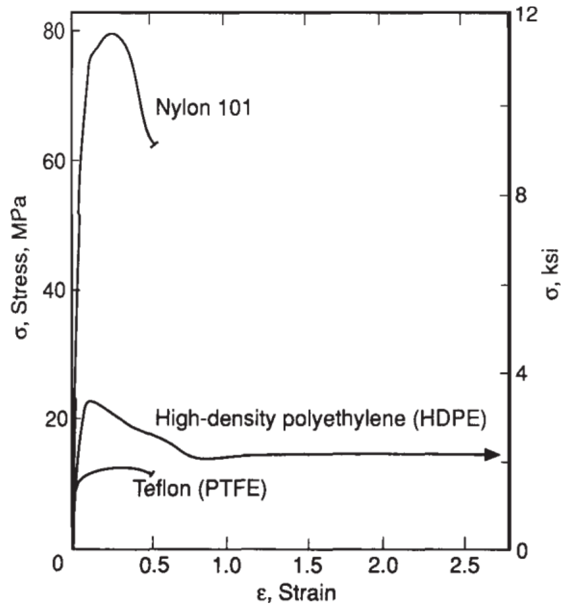


FIGURE 2-22

Tensile Test Stress-Strain Curves of Three Thermoplastic Polymers (From Fig. 5.18, p. 161 in N. E. Dowling, *Mechanical Behavior of Materials*, Prentice-Hall, 1993, with permission)

minum, 8 for steel, and 13 for lead. So, even though the absolute strengths of polymers are low, their specific strengths are respectable due to their low densities.

Polymers are divided into two classes, **thermoplastic** and **thermosets**. **Thermoplastic polymers** can be repeatedly melted and solidified, though their properties can degrade due to the high melt temperatures. Thermoplastics are easy to mold and their rejects or leftovers can be reground and remolded. **Thermosetting polymers** become cross-linked when first heated and will burn, not melt, on reheating. Cross-linking creates connections (like the rungs of a ladder) between the long-chain molecules which wind and twist through a polymer. These cross-connections add strength and stiffness.

Another division among polymers can be made between filled and unfilled compounds. The fillers are usually inorganic materials, such as carbon black, graphite, talc, chopped glass fibers, and metal powders. Fillers are added to both thermoplastic and thermosetting resins, though they are more frequently used in the latter. These filled compounds have superior strength, stiffness, and temperature resistance over that of the raw polymers but are more difficult to mold and to fabricate.

A confusing array of polymers is available commercially. The confusion is increased by a proliferation of brand names for similar compounds made by different manufacturers. The generic chemical names of polymers tend to be long, complex, and hard to remember. In some cases a particular polymer brand name has been so widely used that it has become generic. Nylon, plexiglass, and fiberglass are examples. Learning the generic chemical names and associated brand names of the main families of engineering polymers will eliminate some of the confusion. Table 2-7 shows a number of important polymer families. The mechanical properties of a few of these that have significant engineering applications are included in Appendix A.

Table 2-7
Families of Polymers

Thermoplastics
Cellulosics
Ethylenics
Polyamides
Polyacetals
Polycarbonates
Polyphenyline oxides
Polysulfones
Thermosets
Aminos
Elastomers
Epoxies
Phenolics
Polyesters
Silicones
Urethanes

Ceramics

Ceramic materials are finding increasing application in engineering, and a great deal of effort is being devoted to the development of new ceramic compounds. Ceramics are among the oldest known engineering materials; clay bricks are ceramic materials. Though still widely used in building, clay is not now considered an engineering ceramic. Engineering ceramics are typically compounds of metallic and nonmetallic elements. They may be single oxides of a metal, mixtures of metallic oxides, carbides, borides, nitrides, or other compounds such as Al_2O_3, MgO, SiC, and Si_3N_4, for example. The principal properties of ceramic materials are high hardness and brittleness, high temperature and chemical resistance, high compressive strength, high dielectric strength, and potentially low cost and weight. Ceramic materials are too hard to be machined by conventional techniques and are usually formed by compaction of powder, then fired or sintered to form bonds between particles and increase their strength. The powder compaction can be done in dies or by hydrostatic pressure. Sometimes, glass powder is mixed with the ceramic and the result is fired to melt the glass and fuse the two together. Attempts are being made to replace traditional metals with ceramics in such applications as cast engine blocks, pistons, and other engine parts. The low tensile strength, porosity, and low fracture toughness of most ceramics can be problems in these applications. Plasma-sprayed ceramic compounds are often used as hard coatings on metal substrates to provide wear- and corrosion-resistant surfaces.

Composites

Most composites are man-made, but some, such as wood, occur naturally. Wood is a composite of long cellulose fibers held together in a resinous matrix of lignin. Man-made composites are typically a combination of some strong, fibrous material such as glass, carbon, or boron fibers glued together in a matrix of resin such as epoxy or polyester. The fiberglass material used in boats and other vehicles is a common example of a glass-fiber reinforced polyester (GFRP) composite. The directional material properties of a composite can be tailored to the application by arranging the fibers in different juxtapositions such as parallel, interwoven at random or particular angles, or wound around a mandrel. Custom composites are finding increased use in highly stressed applications such as airframes due to their superior strength-to-weight ratios compared to the common structural metals. Temperature and corrosion resistance can also be designed into some composite materials. These composites are typically neither homogeneous nor isotropic as was discussed in Section 2.3.

It is interesting to note that if one calculates the theoretical strength of any "pure" elemental crystalline material based on the interatomic bonds of the element, the predicted strengths are orders of magnitude larger that those seen in any test of a "real" material, as seen in Table 2-8. The huge differences in actual versus theoretical strengths are attributed to disruptions of the atomic bonds due to crystal defects in the real material. That is, it is considered impossible to manufacture "pure anything" on any realistic superatomic scale. It is presumed that if we could make a "wire" of pure iron only one atom in diameter, it would exhibit its theoretical "super strength." Crystal "whiskers" have been successfully made of some elemental materials and exhibit very high tensile strengths which approach their theoretical values (Table 2-8).

Table 2-8
Iron and Steel Strengths

Form	S_{ut} kpsi (MPa)
Theoretical	2 900 (20*E*3)
Whisker	1 800 (12*E*3)
Fine wire	1 400 (10*E*3)
Mild steel	60 (414)
Cast iron	40 (276)

Other empirical evidence for this theory comes from the fact that fibers of any material made in very small diameters exhibit much higher tensile strengths than would be expected from stress-strain tests of larger samples of the same material. Presumably, the very small cross sections are approaching a "purer" material state. For example, it is well known that glass has poor tensile strength. However, small-diameter glass fibers show much larger tensile strength than sheet glass, making them a practical (and inexpensive) fiber for use in boat hulls, which are subjected to large tensile stresses in use. Small-diameter fibers of carbon and boron exhibit even higher tensile strengths than glass fiber, which explains their use in composites for spacecraft and military aircraft applications, where their relatively high cost is not a barrier.

2.8 SELECTING MATERIALS

One of the most important design decisions is the proper choice of material. Materials limit design and new materials are still being invented that open new design possibilities. It would help if there were a systematic way to select a material for an application. M. F. Ashby has proposed such an approach that plots various material properties against one another to form "materials selection charts."[3] Materials can be roughly divided into six classes, metals, ceramics, polymers (solid or foam), elastomers, glasses, and composites (which include wood). Members of these classes and sub-classes tend to cluster together on a plot of this type.

Figure 2-23 shows such a chart that plots Young's modulus against density, which is called *specific stiffness*. By drawing lines of constant slope on such a chart, one can see which materials possess similar properties. A line of specific stiffness $E / \rho = C$ has been drawn in color on Figure 2-23 and shows that some woods have equivalent specific stiffness to steel and some other metals. The line also passes through the lower range of the engineering composites' "bubble" indicating that fiberglass (GFRP) has about the same specific stiffness as wood and steel, while the nonreinforced thermoplastics such as nylon and polyester have lower specific stiffness. So if you seek the stiffest/lightest material, you want to move up and to the left on the chart. Other lines are shown that have slopes equal to $E^n / \rho = C$ where n is a fraction such as 1/2 or 1/3. These represent loading situations, such as beams in bending, for which the parameter of interest is a nonlinear function of specific stiffness. Since the chart is a log-log plot, exponential functions also plot as straight lines allowing simple comparisons to be made.

Figure 2-24 shows a chart of strength versus density (called *specific strength*) for a number of materials. In this chart, the particular material strength used varies with the material depending on its character. For example, ductile metals and polymers show their yield strength, brittle ceramics their crushing compressive strength, and elastomers their tear strength. The vertical elongation of a material's "bubble" indicates the range of strength values that can obtain due to thermal or work hardening, alloying elements, etc. The colored line drawn on the chart represents a particular value of specific strength or $\sigma / \rho = C$ and shows that the strength-to-weight ratio of some woods are as good as high-strength steel and better than most other metals. It should be no surprise that wood is a popular material in building construction. Note also the high specific strength of engineering ceramics. Unfortunately, their tensile strengths are at best only about 10% of these compressive strengths, which is why you seldom see them used in structures where tensile stresses are commonly encountered.

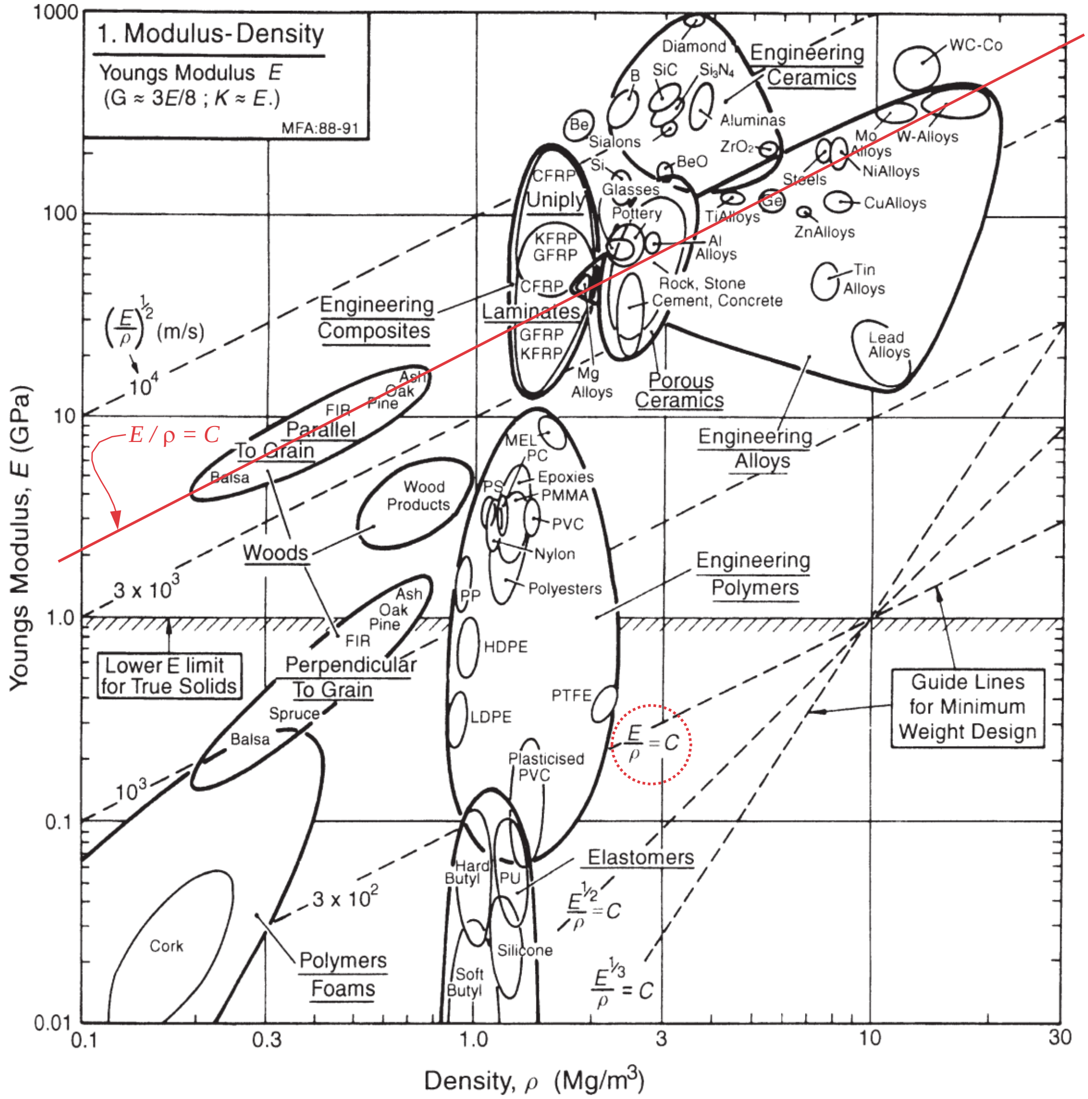


FIGURE 2-23

Young's Modulus Plotted Against Density for Engineering Materials (From Fig. 4-3, p. 37 in M. F. Ashby, *Materials Selection in Mechanical Design*, 2ed, Butterworth-Heinemann 1999, with permission)

Ashby's book[3] is a very useful reference for the practicing engineer. It has dozens of charts of the type shown here that plot various properties against one another in a manner that enhances their comparison and develops good understanding.

2.9 SUMMARY

There are many different kinds of material strengths. It is important to understand which ones are important in particular loading situations. The most commonly measured and

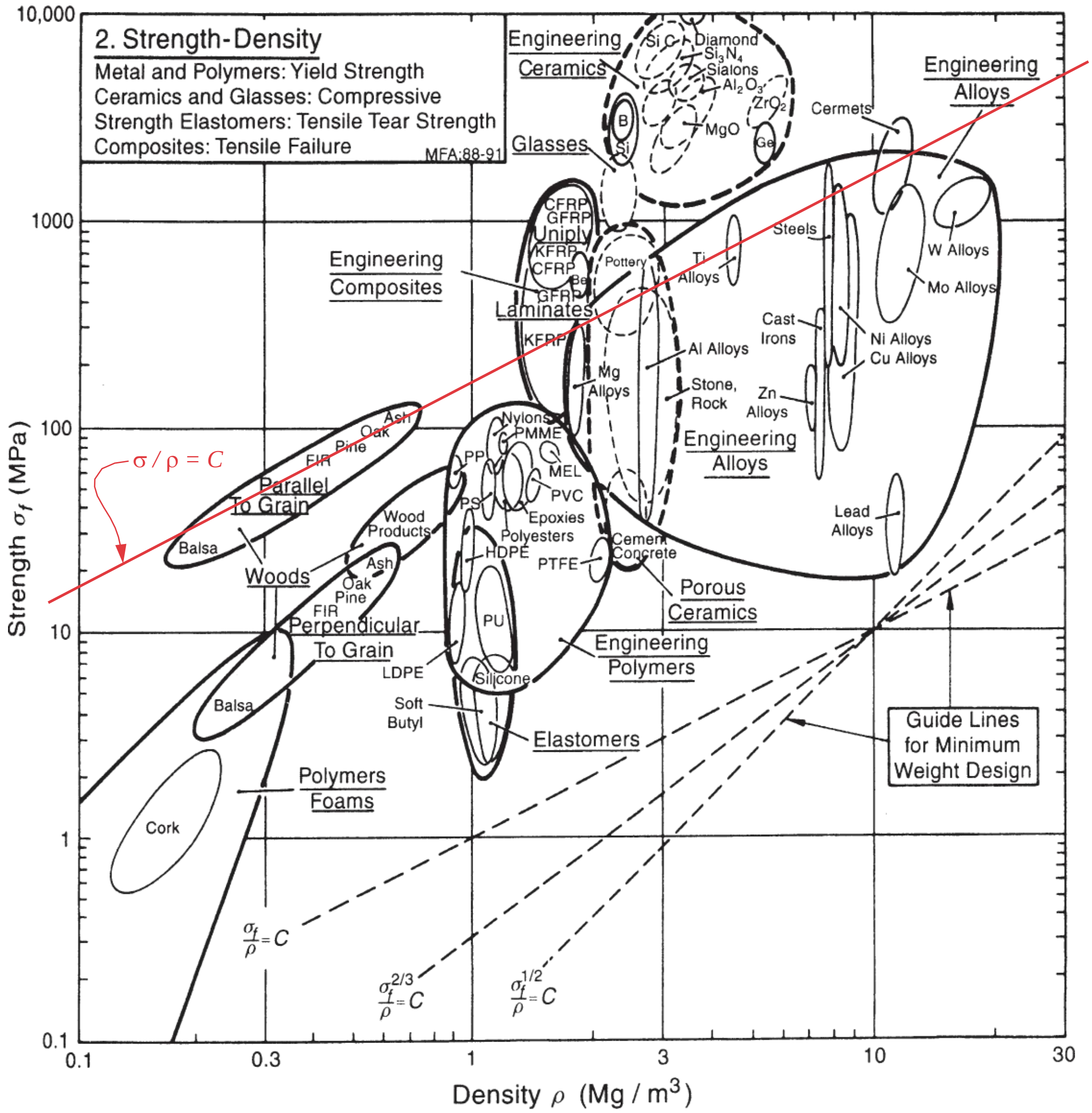


FIGURE 2-24

Strength Plotted Against Density for Engineering Materials (From Fig. 4-4, p. 39 in M. F. Ashby, *Materials Selection in Mechanical Design*, 2ed, Butterworth-Heinemann 1999, with permission)

reported strengths are the **ultimate tensile strength** S_{ut} and the **tensile yield strength** S_y. The S_{ut} indicates the largest stress that the material will accept before fracture, and S_y indicates the stress beyond which the material will take a permanent set. Many materials have **compressive strengths** about equal to their tensile strengths and are called **even materials**. Most wrought metals are in the *even* category. Some materials have significantly different compressive and tensile strengths and these are called **uneven materials**. Cast metals are usually in the *uneven* category, with compressive strengths much greater than their tensile strengths. The **shear strengths** of even materials tend to be about half their tensile strengths, while shear strengths of uneven materials tend to be between their tensile and compressive strengths.

One or more of these strengths may be of interest when the loading is static. If the material is ductile, then S_y is the usual criterion of failure, as a ductile material is capable of significant distortion before fracture. If the material is brittle, as are most cast materials, then the S_{ut} is a more interesting parameter, because the material will fracture before any significant yielding distortion takes place. Yield strength values are nevertheless reported for brittle materials, but are usually calculated based on an arbitrary, small value of strain rather than on any measured yielding of the specimen. Chapter 5 deals with the mechanisms of material failure for both ductile and brittle materials in more detail than does this chapter.

The **tensile test** is the most common measure of these static strength parameters. The **stress-strain curve** (σ–ε) generated in this test is shown in Figure 2-2. The so-called **engineering** σ–ε **curve** differs from the **true** σ–ε **curve** due to the reduction in area of a ductile test specimen during the failure process. Nevertheless, the **engineering** σ–ε **curve** is the standard used to compare materials, since the true σ–ε curve is more difficult to generate.

The slope of the σ–ε curve in the elastic range, called **Young's modulus** or the **modulus of elasticity** *E*, is a very important parameter as it defines the material's stiffness or resistance to elastic deflection under load. If you are designing to control deflections as well as stresses, the value of *E* may be of more interest than the material's strength. While various alloys of a given base material may vary markedly in terms of their strengths, they will have essentially the same *E*. If deflection is the prime concern, a low-strength alloy is as good as a high-strength one of the same base material.

When the loading on the part varies with time it is called **dynamic** or **fatigue loading**. Then the static strengths do not give a good indication of failure. Instead, the **fatigue strength** is of more interest. This strength parameter is measured by subjecting a specimen to dynamic loading until it fails. Both the magnitude of the stress and the number of cycles of stress at failure are reported as the strength criterion. The fatigue strength of a given material will always be lower than its static strength, and often is less than half its S_{ut}. Chapter 6 deals with the phenomenon of fatigue failure of materials in more detail than does this chapter.

Other material parameters of interest to the machine designer are **resilience**, which is the ability to absorb energy without permanent deformation, and **toughness** or the ability to absorb energy without fracturing (but *with* permanent deformation). **Homogeneity** is the uniformity of a material throughout its volume. Many engineering materials, especially metals, can be assumed to be macroscopically homogeneous even though at a microscopic level they are often heterogeneous. **Isotropism** means having properties that are the same regardless of direction within the material. Many engineering materials are reasonably isotropic in the macro and are assumed so for engineering purposes. However, other useful engineering materials such as wood and composites are neither homogeneous nor isotropic and their strengths must be measured separately in different directions. **Hardness** is important in wear resistance and is also related to strength. **Heat treatment**, both through and surface, as well as **cold working** can increase the hardness and strength of some materials.

Important Equations Used in This Chapter

See the referenced sections for information on the proper use of these equations.

Axial tensile stress (Section 2.1):

$$\sigma = \frac{P}{A_o} \tag{2.1a}$$

Axial tensile strain (Section 2.1):

$$\varepsilon = \frac{l - l_o}{l_o} \tag{2.1b}$$

Modulus of elasticity (Young's modulus) (Section 2.1):

$$E = \frac{\sigma}{\varepsilon} \tag{2.2}$$

Modulus of rigidity (Section 2.1):

$$G = \frac{E}{2(1+\nu)} \tag{2.4}$$

Ultimate shear strength (Section 2.1):

$$\begin{aligned} &\text{steels:} && S_{us} \cong 0.80 S_{ut} \\ &\text{other ductile metals:} && S_{us} \cong 0.75 S_{ut} \end{aligned} \tag{2.5b}$$

Shear yield strength (Section 2.1):

$$U = \int_0^{\varepsilon} \sigma \, d\varepsilon \tag{2.6}$$

Modulus of resilience (Section 2.1):

$$U_R \cong \frac{1}{2}\frac{S_y^2}{E} \tag{2.7}$$

Modulus of toughness (Section 2.1):

$$U_T \cong \left(\frac{S_y + S_{ut}}{2} \right) \varepsilon_f \tag{2.8}$$

Arithmetic mean (Section 2.2):

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{2.9b}$$

Standard deviation (Section 2.2):

$$S_d = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu)^2} \tag{2.9c}$$

Ultimate tensile strength as a function of Brinell hardness (Section 2.4):

$$S_{ut} \cong 500\,\mathrm{HB} \pm 30\,\mathrm{HB} \quad \mathrm{psi}$$
$$S_{ut} \cong 3.45\,\mathrm{HB} \pm 0.2\,\mathrm{HB} \quad \mathrm{MPa} \tag{2.10}$$

2.10 REFERENCES

1 **E. B. Haugen and P. H. Wirsching**, "Probabilistic Design." *Machine Design*, **v**. 47, nos. 10-14, Penton Publishing, Cleveland, Ohio, 1975.

2 **H. E. Boyer and T. L. Gall**, eds. *Metals Handbook*. Vol. 1. American Society for Metals: Metals Park, Ohio, 1985.

3 **M. F. Ashby**, *Materials Selection in Mechanical Design*, 2ed., Butterworth and Heinemann, 1999.

2.11 WEB REFERENCES

The web is a useful resource for up-to-date material property information at these and other sites that can be found with a search engine.

http://www.matweb.com

Properties data sheets for over 41,000 metals, plastics, ceramics, and composites.

http://metals.about.com

Material properties and data.

2.12 BIBLIOGRAPHY

For general information on materials, consult the following:

Metals & Alloys in the Unified Numbering System. 6th ed. ASTM/SAE: Philadelphia, Pa., 1994.

Brady, ed. *Materials Handbook*. 13th ed. McGraw-Hill: New York. 1992.

H. E. Boyer, ed. *Atlas of Stress-Strain Curves*. Amer. Soc. for Metals: Metals Park, Ohio, 1987.

K. Budinski, *Engineering Materials: Properties and Selection*. 4th ed. Reston-Prentice-Hall: Reston, Va., 1992.

M. M. Farag, *Selection of Materials and Manufacturing Processes for Engineering Design*. Prentice-Hall International: Hertfordshire, U.K., 1989.

I. Granet, *Modern Materials Science*. Reston-Prentice-Hall: Reston, Va., 1980.

H. W. Pollack, *Materials Science and Metallurgy*. 2nd ed. Reston-Prentice-Hall: Reston, Va., 1977.

S. P. Timoshenko, *History of Strength of Materials*. McGraw-Hill: New York, 1983.

L. H. V. Vlack, *Elements of Material Science and Engineering*. 6th ed. Addison-Wesley: Reading, Mass., 1989.

M. M. Schwartz, ed. *Handbook of Structural Ceramics*. McGraw-Hill: New York. 1984.

For specific information on material properties, consult the following:

H. E. Boyer and T. L. Gall, ed. *Metals Handbook*. Vol. 1. American Society for Metals: Metals Park, Ohio, 1985.

U. S. Department of Defense. *Metallic Materials and Elements for Aerospace Vehicles and Structures* MIL-HDBK-5H, 1998.

R. Juran, ed. *Modern Plastics Encyclopedia*. McGraw-Hill: New York, 1988.

J. D. Lubahn and R. P. Felgar, *Plasticity and Creep of Metals*. Wiley: New York, 1961.

For information on failure of materials, consult the following:

J. A. Collins, *Failure of Materials in Mechanical Design*. Wiley: New York, 1981.

N. E. Dowling, *Mechanical Behavior of Materials*. Prentice-Hall: Englewood Cliffs, N.J., 1992.

R. C. Juvinall, *Stress, Strain and Strength*. McGraw-Hill: New York, 1967.

For information on plastics and composites, consult the following:

ASM, *Engineered Materials Handbook: Composites*. Vol. 1. American Society for Metals: Metals Park, Ohio, 1987.

ASM, *Engineered Materials Handbook: Engineering Plastics*. Vol. 2. American Society for Metals: Metals Park, Ohio, 1988.

Harper, ed. *Handbook of Plastics, Elastomers and Composites*. 2nd ed. McGraw-Hill: New York, 1990.

J. E. Hauck, "Long-Term Performance of Plastics." *Materials in Design Engineering*, pp. 113-128, November, 1965.

M. M. Schwartz, *Composite Materials Handbook*. McGraw-Hill: New York, 1984.

For information on manufacturing processes, see:

R. W. Bolz, *Production Processes: The Productivity Handbook*. Industrial Press: New York, 1974.

J. A. Schey, *Introduction to Manufacturing Processes*. McGraw-Hill: New York, 1977.

2.13 PROBLEMS

2-1 Figure P2-1 shows stress-strain curves for three failed tensile-test specimens. All are plotted on the same scale.

(a) Characterize each material as brittle or ductile.
(b) Which is the stiffest?
(c) Which has the highest ultimate strength?
(d) Which has the largest modulus of resilience?
(e) Which has the largest modulus of toughness?

2-2 Determine an approximate ratio between the yield strength and ultimate strength for each material shown in Figure P2-1.

2-3 Which of the steel alloys shown in Figure 2-19 would you choose to obtain

(a) Maximum strength
(b) Maximum modulus of resilience
(c) Maximum modulus of toughness
(d) Maximum stiffness

Table P2-0
Topic/Problem Matrix

2.1 Material Properties

2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 2-8, 2-9, 2-10, 2-11, 2-12, 2-18, 2-19, 2-20, 2-21, 2-22, 2-23

2.4 Hardness

2-13, 2-14

2.6 General Properties

2-15, 2-16, 2-17, 2-24, 2-25, 2-26

2.8 Selecting Materials

2-37, 2-38, 2-39, 2-40

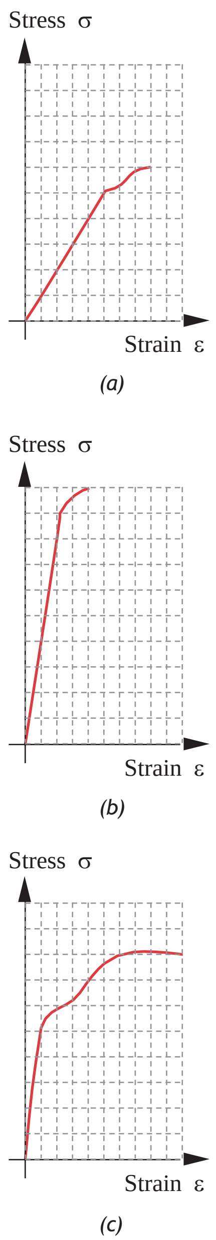


FIGURE P2-1

Stress-Strain Curves

2-4 Which of the aluminum alloys shown in Figure 2-21 would you choose to obtain

(a) Maximum strength
(b) Maximum modulus of resilience
(c) Maximum modulus of toughness
(d) Maximum stiffness

2-5 Which of the thermoplastic polymers shown in Figure 2-22 would you choose in order to obtain

(a) Maximum strength
(b) Maximum modulus of resilience
(c) Maximum modulus of toughness
(d) Maximum stiffness

*2-6 A metal has a strength of 414 MPa at its elastic limit and the strain at that point is 0.002. Assume the test specimen is 12.8-mm dia and has a 50-mm gage length. What is its modulus of elasticity? What is the strain energy at the elastic limit? Can you define the type of metal based on the given data?

2-7 A metal has a strength of 41.2 kpsi (284 MPa) at its elastic limit and the strain at that point is 0.004. Assume the test specimen is 0.505-in dia and has a 2-in gage length. What is the strain energy at the elastic limit? Can you define the type of metal based on the given data?

*2-8 A metal has a strength of 134 MPa at its elastic limit and the strain at that point is 0.003. What is its modulus of elasticity? Assume the test specimen is 12.8-mm dia and has a 50-mm gage length. What is its modulus of elasticity? What is the strain energy at the elastic limit? Can you define the type of metal based on the given data?

*2-9 A metal has a strength of 100 kpsi (689 MPa) at its elastic limit and the strain at that point is 0.006. What is its modulus of elasticity? What is the strain energy at the elastic limit? Assume the test specimen is 0.505-in dia and has a 2-in gage length. Can you define the type of metal based on the given data?

2-10 A material has a yield strength of 689 MPa at an offset of 0.6% strain. What is its modulus of resilience?

2-11 A material has a yield strength of 60 kpsi (414 MPa) at an offset of 0.2% strain. What is its modulus of resilience?

*2-12 A steel has a yield strength of 414 MPa, an ultimate tensile strength of 689 MPa, and an elongation at fracture of 15%. What is its approximate modulus of toughness? What is its approximate modulus of resilience?

2-13 The Brinell hardness of a steel specimen was measured to be 250 HB. What is the material's approximate tensile strength? What is its hardness on the Vickers scale? The Rockwell scale?

*2-14 The Brinell hardness of a steel specimen was measured to be 340 HB. What is the material's approximate tensile strength? What is its hardness on the Vickers scale? The Rockwell scale?

2-15 What are the principal alloy elements of an AISI 4340 steel? How much carbon does it have? Is it hardenable? By what techniques?

*2-16 What are the principal alloy elements of an AISI 1095 steel? How much carbon does it have? Is it hardenable? By what techniques?

2-17 What are the principal alloy elements of an AISI 6180 steel? How much carbon does it have? Is it hardenable? By what techniques?

2-18 Which of the steels in Problems 2-15, 2-16, and 2-17 is the stiffest?

* Answers to these problems are provided in Appendix D.

2-19 Calculate the *specific strength* and *specific stiffness* of the following materials and pick one for use in an aircraft wing spar.

(a) Steel S_{ut} = 80 kpsi (552 MPa)
(b) Aluminum S_{ut} = 60 kpsi (414 MPa)
(c) Titanium S_{ut} = 90 kpsi (621 MPa)

2-20 If maximum *impact resistance* were desired in a part, which material properties would you look for?

2-21 Refer to the tables of material data in Appendix A and determine the strength-to-weight ratios of the following material alloys based on their tensile yield strengths: heat-treated 2024 aluminum, SAE 1040 cold-rolled steel, Ti-75A titanium, type 302 cold-rolled stainless steel.

2-22 Refer to the tables of material data in Appendix A and determine the strength-to-weight ratios of the following material alloys based on their ultimate tensile strengths: heat-treated 2024 aluminum, SAE 1040 cold-rolled steel, unfilled acetal plastic, Ti-75A titanium, type 302 cold-rolled stainless steel.

2-23 Refer to the tables of material data in Appendix A and calculate the specific stiffnesses of aluminum, titanium, gray cast iron, ductile iron, bronze, carbon steel, and stainless steel. Rank them in increasing order of this property and discuss the engineering significance of these data.

2-24 Call your local steel and aluminum distributors (consult the Yellow Pages) and obtain current costs per pound for round stock of consistent size in low-carbon (SAE 1020) steel, SAE 4340 steel, 2024-T4 aluminum, and 6061-T6 aluminum. Calculate a strength/dollar ratio and a stiffness/dollar ratio for each alloy. Which would be your first choice on a cost-efficiency basis for an axial-tension-loaded round rod

(a) If maximum strength were needed?
(b) If maximum stiffness were needed?

2-25 Call your local plastic stock-shapes distributors (consult the Yellow Pages) and obtain current costs per pound for round rod or tubing of consistent size in plexiglass, acetal, nylon 6/6, and PVC. Calculate a strength/dollar ratio and a stiffness/dollar ratio for each alloy. Which would be your first choice on a cost-efficiency basis for an axial-tension-loaded round rod or tube of particular diameters. (Note: material parameters can be found in Appendix A.)

(a) If maximum strength were needed?
(b) If maximum stiffness were needed?

2-26 A part has been designed and its dimensions cannot be changed. To minimize its deflections under the same loading in all directions irrespective of stress levels, which of these materials would you choose: aluminum, titanium, steel, or stainless steel? Why?

*2-27 Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the tensile yield strength for 1050 steel quenched and tempered at 400F if a reliability of 99.9% is required?

2-28 Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the ultimate tensile strength for 4340 steel quenched and tempered at 800F if a reliability of 99.99% is required?

2-29 Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the ultimate tensile strength for 4130 steel quenched and tempered at 400F if a reliability of 90% is required?

2-30 Assuming that the mechanical properties data given in Appendix Table A-9 for some carbon steels represents mean values, what is the value of the tensile yield strength for 4140 steel quenched and tempered at 800F if a reliability of 99.999% is required?

* Answers to these problems are provided in Appendix D.

2-31 A steel part is to be plated to give it better corrosion resistance. Two materials are being considered: cadmium and nickel. Considering only the problem of galvanic action, which would you choose? Why?

2-32 A steel part with many holes and sharp corners is to be plated with nickel. Two processes are being considered: electroplating and electroless plating. Which process would you choose? Why?

2-33 What is the common treatment used on aluminum to prevent oxidation? What other metals can also be treated with this method? What options are available with this method?

*2-34 Steel is often plated with a less noble metal that acts as a sacrificial anode that will corrode instead of the steel. What metal is commonly used for this purpose (when the finished product will not be exposed to saltwater), what is the coating process called, and what are the common processes used to obtain the finished product?

2-35 A low-carbon steel part is to be heat-treated to increase its strength. If an ultimate tensile strength of approximately 550 MPa is required, what mean Brinell hardness should the part have after treatment? What is the equivalent hardness on the Rockwell scale?

2-36 A low-carbon steel part has been tested for hardness using the Brinell method and is found to have a hardness of 220 HB. What are the approximate lower and upper limits of the ultimate tensile strength of this part in MPa?

2-37 Figure 2-24 shows "guide lines" for minimum weight design when failure is the criterion. The guide line, or index, for minimizing the weight of a beam in bending is $\sigma_f^{2/3} / \rho$, where σ_f is the yield strength of a material and ρ is its mass density. For a given cross-section shape the weight of a beam with given loading will be minimized when this index is maximized. The following materials are being considered for a beam application: 5052 aluminum, cold rolled; CA-170 beryllium copper, hard plus aged; and 4130 steel, Q&T @ 1200F. The use of which of these three materials will result in the least-weight beam?

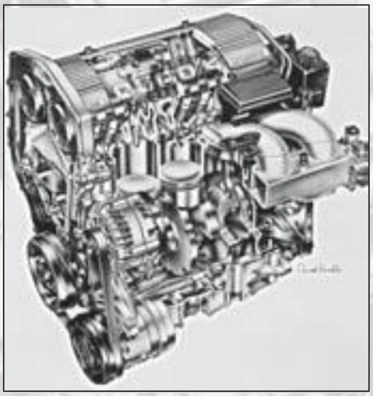
2-38 Figure 2-24 shows "guide lines" for minimum weight design when failure is the criterion. The guide line, or index, for minimizing the weight of a member in tension is σ_f / ρ, where σ_f is the yield strength of a material and ρ is its mass density. The weight of a member with given loading will be minimized when this index is maximized. For the three materials given in Problem 2-37, which will result in the lowest weight tension member?

2-39 Figure 2-23 shows "guide lines" for minimum weight design when stiffness is the criterion. The guide line, or index, for minimizing the weight of a beam in bending is $E^{1/2} / \rho$, where E is the modulus of elasticity of a material and ρ is its mass density. For a given cross-section shape the weight of a beam with given stiffness will be minimized when this index is maximized. The following materials are being considered for a beam application: 5052 aluminum, cold rolled; CA-170 beryllium copper, hard plus aged; and 4130 steel, Q&T @ 1200F. The use of which of these three materials will result in the lowest-weight beam?

2-40 Figure 2-24 shows "guide lines" for minimum weight design when stiffness is the criterion. The guide line, or index, for minimizing the weight of a member in tension is E / ρ, where E is the modulus of elasticity of a material and ρ is its mass density. The weight of a member with given stiffness will be minimized when this index is maximized. For the three materials given in Problem 2-39, which will result in the lowest-weight tension member?

* Answers to these problems are provided in Appendix D.

3



LOAD DETERMINATION

If a builder has built a house for a man and his work is not strong and the house falls in and kills the householder, that builder shall be slain.

FROM THE CODE OF HAMMURABI, 2150 BC

3.0 INTRODUCTION

This chapter provides a review of the fundamentals of static and dynamic force analysis, impact forces, and beam loading. The reader is assumed to have had first courses in statics and dynamics. Thus, this chapter presents only a brief, general overview of those topics but also provides more powerful solution techniques, such as the use of singularity functions for beam calculations. The Newtonian solution method of force analysis is reviewed and a number of case-study examples are presented to reinforce understanding of this subject. The case studies also set the stage for analysis of these same systems for stress, deflection, and failure modes in later chapters.

Table 3-0 shows the variables used in this chapter and references the equations, sections, or case studies in which they are used. At the end of the chapter, a summary section is provided which groups all the significant equations from this chapter for easy reference and identifies the chapter section in which their discussion can be found.

3.1 LOADING CLASSES

The type of loading on a system can be divided into several classes based on the character of the applied loads and the presence or absence of system motion. Once the general configuration of a mechanical system is defined and its kinematic motions calculated, the next task is to determine the magnitudes and directions of all the forces and couples present on the various elements. These loads may be constant or may be varying over time. The elements in the system may be stationary or moving. The most general class is that of a moving system with time-varying loads. The other combinations are subsets of the general class.

3

Table 3-0 Variables Used in This Chapter

Symbol	Variable	ips units	SI units	See
a	distance to load	in	m	Sect. 3.9
b	distance to load	in	m	Sect. 3.9
d	damping	lb-sec/in	N-sec/m	Eq. 3.6
E	energy	in-lb	joules	Eq. 3.9, 3.10
F	force or load	lb	N	Sect. 3.3
f_d	damped natural frequency	Hz	Hz	Eq. 3.7
f_n	natural frequency	Hz	Hz	Eq. 3.4
g	gravitational acceleration	in/sec^2	m/sec^2	Eq. 3.12
I_x	mass moment of inertia about *x* axis	lb-in-sec^2	kg-m^2	Sect. 3.3
I_y	mass moment of inertia about *y* axis	lb-in-sec^2	kg-m^2	Sect. 3.3
I_z	mass moment of inertia about *z* axis	lb-in-sec^2	kg-m^2	Sect. 3.3
k	spring rate or spring constant	lb/in	N/m	Eq. 3.5
l	length	in	m	Sect. 3.9
m	mass	lb-sec^2/in	kg	Sect. 3.3
N	normal force	in	m	Case 4A
M	moment, moment function	lb-in	N-m	Sect. 3.3, 3.9
q	beam loading function	lb	N	Sect. 3.9
$\mathbf{R}$	position vector	in	m	Sect. 3.4
R	reaction force	lb	N	Sect. 3.9
v	linear velocity	in/sec	m/sec	Eq. 3.10
V	beam shear function	lb	N	Sect. 3.9
W	weight	lb	N	Eq. 3.14
x	generalized length variable	in	m	Sect. 3.9
y	displacement	in	m	Eq. 3.5, 3.8
δ	deflection	in	m	Eq. 3.5
η	correction factor	none	none	Eq. 3.10
μ	coefficient of friction	none	none	Case 4A
ω	rotational or angular velocity	rad/sec	rad/sec	Case 5A
ω_d	damped natural frequency	rad/sec	rad/sec	Eq. 3.7
ω_n	natural frequency	rad/sec	rad/sec	Eq. 3.4

Table 3-1 shows the four possible classes. Class 1 is a stationary system with constant loads. One example of a Class 1 system is the base frame for an arbor press used in a machine shop. The base is required to support the dead weight of the arbor press which is essentially constant over time, and the base frame does not move. The parts brought to the arbor press (to have something pressed into them) temporarily add their weight to the load on the base, but this is usually a small percentage of the dead weight. A static load analysis is all that is necessary for a Class 1 system.

Title-page photograph courtesy of Chevrolet Division of General Motors Co., Detroit, Mich.

Table 3-1 Load Classes

	Constant Loads	Time-Varying Loads
Stationary Elements	Class 1	Class 2
Moving Elements	Class 3	Class 4

Class 2 describes a stationary system with time-varying loads. An example is a bridge which, though essentially stationary, is subjected to changing loads as vehicles drive over it and wind impinges on its structure. Class 3 defines a moving system with constant loads. Even though the applied external loads may be constant, any significant accelerations of the moving members can create time-varying reaction forces. An example might be a powered rotary lawn mower. Except for the case of mowing the occasional rock, the blades experience a nearly constant external load from mowing the grass. However, the accelerations of the spinning blades can create high loads at their fastenings. A dynamic load analysis is necessary for Classes 2 and 3.

Note however that, if the motions of a Class 3 system are so slow as to generate negligible accelerations on its members, it could qualify as a Class 1 system and then would be called *quasi-static*. An automobile scissors jack (see Figure 3-5, p. 88) can be considered to be a Class 1 system since the external load (when used) is essentially constant, and the motions of the links are slow with negligible accelerations. The only complexity introduced by the motions of the elements in this example is that of determining in which position the internal loads on the jack's elements will be maximal, since they vary as the jack is raised, despite the essentially constant external load.

Class 4 describes the general case of a rapidly moving system subjected to time-varying loads. Note that even if the applied external loads are essentially constant in a given case, the dynamic loads developed on the elements from their accelerations will still vary with time. Most machinery, especially if powered by a motor or engine, will be in Class 4. An example of such a system is the engine in your car. The internal parts (crankshaft, connecting rods, pistons, etc.) are subjected to time-varying loads from the gasoline explosions, and also experience time-varying inertial loads from their own accelerations. A dynamic load analysis is necessary for Class 4.

3.2 FREE-BODY DIAGRAMS

In order to correctly identify all potential forces and moments on a system, it is necessary to draw accurate free-body diagrams (FBDs) of each member of the system. These FBDs should show a general shape of the part and display all the forces and moments that are acting on it. There may be external forces and moments applied to the part from outside the system, and there will be interconnection forces and/or moments where each part joins or contacts adjacent parts in the assembly or system.

In addition to the known and unknown forces and couples shown on the FBD, the dimensions and angles of the elements in the system are defined with respect to local coordinate systems located at the **centers of gravity** (CG) of each element.* For a dynamic load analysis, the kinematic accelerations, both angular and linear (at the CG), need to be known or calculated for each element prior to doing the load analysis.

* While it is not a requirement that the local coordinate system for each element be located at its CG, this approach provides consistency and simplifies the dynamic calculations. Further, most solid modeling CAD/CAE systems will automatically calculate the mass properties of parts with respect to their CGs. The approach taken here is to apply a consistent method that works for both static and dynamic problems and that is also amenable to computer solution.

3

3.3 LOAD ANALYSIS

This section presents a brief review of Newton's laws and Euler's equations as applied to dynamically loaded and statically loaded systems in both 3-D and 2-D. The method of solution presented here may be somewhat different than that used in your previous statics and dynamics courses. The approach taken here in setting up the equations for force and moment analysis is designed to facilitate computer programming of the solution.

This approach assumes all *unknown* forces and moments on the system to be positive in sign, regardless of what one's intuition or an inspection of the free-body diagram might indicate as to their probable directions. However, all *known* force components are given their proper signs to define their directions. The simultaneous solution of the set of equations that results will cause all the unknown components to have the proper signs when the solution is complete. This is ultimately a simpler approach than the one often taught in statics and dynamics courses which requires that the student assume directions for all unknown forces and moments (a practice that does help the student develop some intuition, however). Even with that traditional approach, an incorrect assumption of direction results in a sign reversal on that component in the solution. Assuming all unknown forces and moments to be positive allows the resulting computer program to be simpler than would otherwise be the case. The simultaneous equation solution method used is extremely simple in concept, though it requires the aid of a computer to solve. Software is provided with the text to solve the simultaneous equations. See program MATRIX on the CD-ROM.

Real dynamic systems are three dimensional and thus must be analyzed as such. However, many 3-D systems can be analyzed by simpler 2-D methods. Accordingly, we will investigate both approaches.

Three-Dimensional Analysis

Since three of the four cases potentially require dynamic load analysis, and because a static force analysis is really just a variation on the dynamic analysis, it makes sense to start with the dynamic case. Dynamic load analysis can be done by any of several methods, but the one that gives the most information about internal forces is the Newtonian approach based on Newton's laws.

NEWTON'S FIRST LAW *A body at rest tends to remain at rest and a body in motion at constant velocity will tend to maintain that velocity unless acted upon by an external force.*

NEWTON'S SECOND LAW *The time rate of change of momentum of a body is equal to the magnitude of the applied force and acts in the direction of the force*.

Newton's second law can be written for a rigid body in two forms, one for linear forces and one for moments or torques:

$$\sum \mathbf{F} = m\mathbf{a} \qquad\qquad \sum \mathbf{M}_G = \dot{\mathbf{H}}_G \tag{3.1a}$$

where $\mathbf{F}$ = force, m = mass, $\mathbf{a}$ = acceleration, $\mathbf{M}_G$ = moment about the center of gravity, and $\dot{\mathbf{H}}_G$ = the time rate of change of the moment of momentum, or the angular momentum about the CG. The left sides of these equations respectively sum all the forces and moments that act on the body, whether from known applied forces or from interconnections with adjacent bodies in the system.

For a three-dimensional system of connected rigid bodies, this vector equation for the linear forces can be written as three scalar equations involving orthogonal components taken along a local *x*, *y*, *z* axis system with its origin at the CG of the body:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \tag{3.1b}$$

If the *x*, *y*, *z* axes are chosen coincident with the principal axes of inertia of the body,* the angular momentum of the body is defined as

$$\mathbf{H}_G = I_x\omega_x\hat{\mathbf{i}} + I_y\omega_y\hat{\mathbf{j}} + I_z\omega_z\hat{\mathbf{k}} \tag{3.1c}$$

where I_x, I_y, and I_z are the principal centroidal mass moments of inertia (second moments of mass) about the principal axes. This vector equation can be substituted into equation 3.1*a* to yield the three scalar equations known as **Euler's equations**:

$$\begin{aligned}
\sum M_x &= I_x\alpha_x - \left(I_y - I_z\right)\omega_y\omega_z \\
\sum M_y &= I_y\alpha_y - \left(I_z - I_x\right)\omega_z\omega_x \\
\sum M_z &= I_z\alpha_z - \left(I_x - I_y\right)\omega_x\omega_y
\end{aligned} \tag{3.1d}$$

where M_x, M_y, M_z are moments about those axes and α_x, α_y, α_z are the angular accelerations about the axes. This assumes that the inertia terms remain constant with time, i.e., the mass distribution about the axes is constant.

NEWTON'S THIRD LAW states that *when two particles interact, a pair of equal and opposite reaction forces will exist at their contact point. This force pair will have the same magnitude and act along the same direction line, but have opposite sense*.

We will need to apply this relationship as well as applying the second law in order to solve for the forces on assemblies of elements that act upon one another. The six equations in equations 3.1*b* and 3.1*d* can be written for each rigid body in a 3-D system. In addition, as many (third-law) reaction force equations as are necessary will be written and the resulting set of equations solved simultaneously for the forces and moments. The number of second-law equations will be up to six times the number of individual parts in a three-dimensional system (plus the reaction equations), meaning that even simple systems result in large sets of simultaneous equations. A computer is needed to solve these equations, though high-end pocket calculators will solve large sets of simultaneous equations also. The reaction (third-law) equations are often substituted into the second-law equations to reduce the total number of equations to be solved simultaneously.

Two-Dimensional Analysis

All real machines exist in three dimensions but many three-dimensional systems can be analyzed two dimensionally if their motions exist only in one plane or in parallel planes.

* This is a convenient choice for symmetric bodies but may be less convenient for other shapes. See F. P. Beer and E. R. Johnson, *Vector Mechanics for Engineers*, 3rd ed., 1977, McGraw-Hill, New York, Chap. 18, "Kinetics of Rigid Bodies in Three Dimensions."

Euler's equations 3.1*d* show that if the rotational motions (ω, α) and applied moments or couples exist about only one axis (say the z axis), then that set of three equations reduces to one equation,

$$\sum M_z = I_z \alpha_z \tag{3.2a}$$

because the ω and α terms about the x and y axes are now zero. Equation 3.1*b* is reduced to

$$\sum F_x = m a_x \qquad \sum F_y = m a_y \tag{3.2b}$$

Equations 3.2 can be written for all the connected bodies in a two-dimensional system and the entire set solved simultaneously for forces and moments. The number of second-law equations will now be up to three times the number of elements in the system plus the necessary reaction equations at connecting points, again resulting in large systems of equations for even simple systems. Note that even though all motion is about one (z) axis in a 2-D system, there may still be loading components in the z direction due to external forces or couples.

Static Load Analysis

The difference between a dynamic loading situation and a static one is the presence or absence of accelerations. If the accelerations in equations 3.1 and 3.2 are all zero, then for the three-dimensional case these equations reduce to

$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array} \tag{3.3a}$$

and for the two-dimensional case,

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0 \tag{3.3b}$$

Thus, we can see that the static loading situation is just a special case of the dynamic loading one, in which the accelerations happen to be zero. A solution approach based on the dynamic case will then also satisfy the static one with appropriate substitutions of zero values for the absent accelerations.

3.4 TWO-DIMENSIONAL, STATIC LOADING CASE STUDIES

This section presents a series of three case studies of increasing complexity, all limited to two-dimensional static loading situations. A bicycle handbrake lever, a crimping tool, and a scissors jack are the systems analyzed. These case studies provide examples of the simplest form of force analysis, having no significant accelerations and having forces acting in only two dimensions.

CASE STUDY 1A

Bicycle Brake Lever Loading Analysis

Problem: **Determine the forces on the elements of the bicycle brake lever assembly shown in Figure 3-1 during braking.**

Given: **The geometry of each element is known. The average human's hand can develop a grip force of about 267 N (60 lb) in the lever position shown.**

Assumptions: **The accelerations are negligible. All forces are coplanar and two dimensional. A Class I load model is appropriate and a static analysis is acceptable.**

Solution: **See Figures 3-1, 3-2, and Table 3-2, parts 1 and 2.**

1 Figure 3-1 shows the handbrake lever assembly, which consists of three subassemblies: the handlebar (1), the lever (2), and the cable (3). The lever is pivoted to the handlebar and the cable is connected to the lever. The cable runs within a plastic-lined sheath (for low friction) down to the brake caliper assembly at the bicycle's wheel rim. The sheath provides a compressive force to balance the tension in the cable ($F_{sheath} = -F_{cable}$). The user's hand applies equal and opposite forces at some points on the lever and handgrip. These forces are transformed to a larger force in the cable by the lever ratio of part 2.

Figure 3-1 is a free-body diagram of the entire assembly since it shows all the forces and moments potentially acting on it except for its weight, which is small compared to the applied forces and is thus neglected for this analysis. The "broken away" portion of the handlebar can provide *x* and *y* force components and a moment if required for equilibrium. These reaction forces and moments are arbitrarily shown as positive in sign. Their actual signs will "come out in the wash" in the calculations. The known applied forces are shown acting in their actual directions and senses.

2 Figure 3-2 shows the three subassembly elements separated and drawn as free-body diagrams with all relevant forces and moments applied to each element, again neglect-

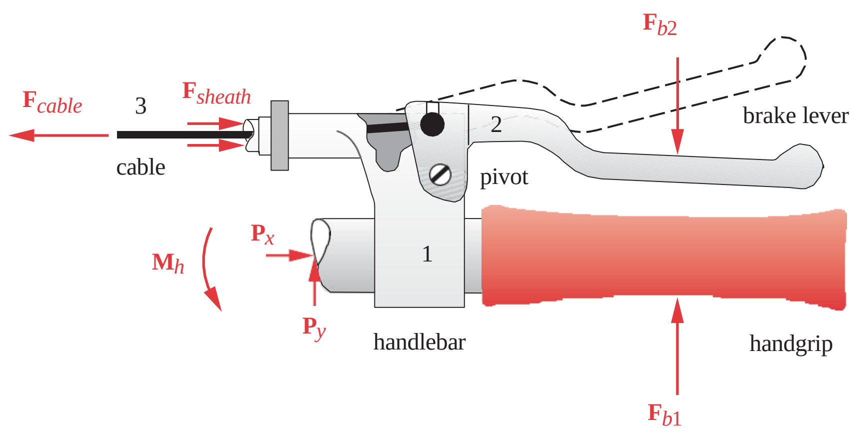


FIGURE 3-1

Bicycle Brake Lever Assembly

Table 3–2 - part I
Case Study 1A
Given Data

Variable	Value	Unit
F_{13x}	0.0	N
F_{b2x}	0.0	N
F_{b2y}	-267.0	N
θ	184.0	deg
ϕ	180.0	deg
R_{b2x}	39.39	mm
R_{b2y}	2.07	mm
R_{32x}	-50.91	mm
R_{32y}	4.66	mm
R_{12x}	-47.91	mm
R_{12y}	-7.34	mm
R_{21x}	7.0	mm
R_{21y}	19.0	mm
R_{b1x}	47.5	mm
R_{b1y}	-14.0	mm
R_{31x}	-27.0	mm
R_{31y}	30.0	mm
R_{px}	-27.0	mm
R_{py}	0.0	mm
R_{dx}	-41.0	mm
R_{dy}	27.0	mm

ing the weights of the parts. The lever (part 2) has three forces on it, $\mathbf{F}_{b2}$, $\mathbf{F}_{32}$, and $\mathbf{F}_{12}$. The two-character subscript notation used here should be read as, force of element 1 **on** 2 ($\mathbf{F}_{12}$) or force **at** *B* on 2 ($\mathbf{F}_{b2}$), etc. This defines the source of the force (first subscript) and the element on which it acts (second subscript).

This notation will be used consistently throughout this text for both forces and position vectors such as $\mathbf{R}_{b2}$, $\mathbf{R}_{32}$, and $\mathbf{R}_{12}$ in Figure 3-2 which serve to locate the above three forces in a local, nonrotating coordinate system whose origin is at the center of gravity (CG) of the element or subassembly being analyzed.*

On this brake lever, $\mathbf{F}_{b2}$ is an applied force whose magnitude and direction are known. $\mathbf{F}_{32}$ is the force in the cable. Its direction is known but not its magnitude. Force $\mathbf{F}_{12}$ is provided by part 1 on part 2 at the pivot pin. Its magnitude and direction are both unknown. We can write equations 3.3*b* for this element to sum forces in the *x* and *y* directions and sum moments about the CG. Note that all unknown forces and moments are initially assumed positive in the equations. Their true signs will come out in the calculation.† However, all known or given forces must carry their proper signs.

$$\sum F_x = F_{12x} + F_{b2x} + F_{32x} = 0$$
$$\sum F_y = F_{12y} + F_{b2y} + F_{32y} = 0 \qquad (a)$$
$$\sum \mathbf{M}_z = \left(\mathbf{R}_{12} \times \mathbf{F}_{12}\right) + \left(\mathbf{R}_{b2} \times \mathbf{F}_{b2}\right) + \left(\mathbf{R}_{32} \times \mathbf{F}_{32}\right) = 0$$

The cross products in the moment equation represent the "turning forces" or moments created by the application of these forces at points remote from the CG of the element. Recall that these cross products can be expanded to

$$\sum M_z = \left(R_{12x}F_{12y} - R_{12y}F_{12x}\right) + \left(R_{b2x}F_{b2y} - R_{b2y}F_{b2x}\right)$$
$$+\left(R_{32x}F_{32y} - R_{32y}F_{32x}\right) = 0 \qquad (b)$$

We have three equations and four unknowns (F_{12x}, F_{12y}, F_{32x}, F_{32y}) at this point, so we need another equation. It is available from the fact that the direction of $\mathbf{F}_{32}$ is known. (The cable can pull only along its axis.) We can express one component of the cable force $\mathbf{F}_{32}$ in terms of its other component and the known angle, θ of the cable.

$$F_{32y} = F_{32x}\tan\theta \qquad (c)$$

We could now solve the four unknowns for this element, but will wait to do so until the equations for the other two links are defined.

3 Part 3 in Figure 3-2 is the cable which passes through a hole in part 1. This hole is lined with a low-friction material which allows us to assume no friction at the joint between parts 1 and 3. We will further assume that the three forces $\mathbf{F}_{13}$, $\mathbf{F}_{23}$, and $\mathbf{F}_{cable}$ form a concurrent system of forces acting through the CG and thus create no moment. With this assumption only a summation of forces is necessary for this element.

$$\sum F_x = F_{cable_x} + F_{13x} + F_{23x} = 0$$
$$\sum F_y = F_{cable_y} + F_{13y} + F_{23y} = 0 \qquad (d)$$

4 The assembly of elements labeled part 1 in Figure 3-2 can have both forces and moments on it (i.e., it is not a concurrent system), so the three equations 3.3*b* are needed.

* Actually, for a simple static analysis such as the one in this example, any point (on or off the element) can be taken as the origin of the local coordinate system. However, in a dynamic force analysis it simplifies the analysis if the coordinate system is placed at the CG. So, for the sake of consistency, and to prepare for the more complicated dynamic analysis problems ahead, we will use the CG as the origin even in the static cases here.

† You may not have done this in your statics class but this approach makes the problem more amenable to a computer solution. Note that regardless of the direction shown for any unknown force on the FBD, we will assume its components to be positive in the equations. The angles of the known (given) forces (or the signs of their components) do have to be correctly input to the equations, however.

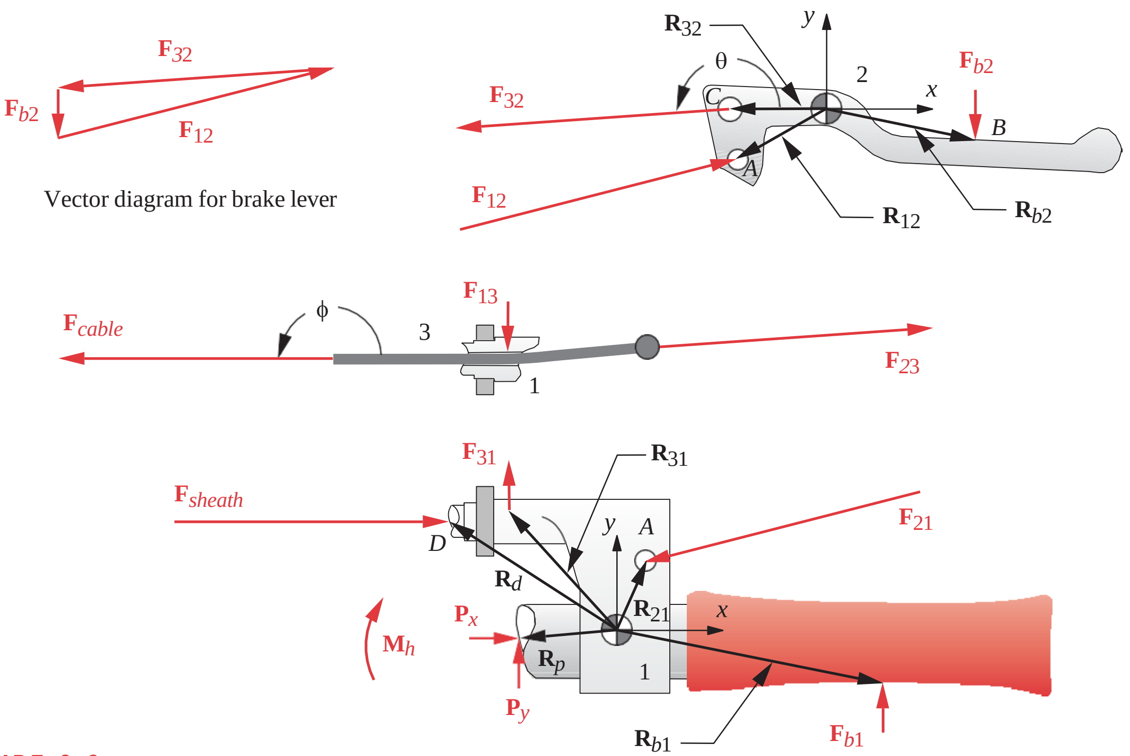


FIGURE 3-2

Bicycle Brake Lever Free-Body Diagrams

$$\sum F_x = F_{21x} + F_{b1x} + F_{31x} + P_x + F_{sheath_x} = 0$$

$$\sum F_y = F_{21y} + F_{b1y} + F_{31y} + P_y = 0 \qquad (e)$$

$$\sum \mathbf{M}_z = \mathbf{M}_h + \left(\mathbf{R}_{21} \times \mathbf{F}_{21}\right) + \left(\mathbf{R}_{b1} \times \mathbf{F}_{b1}\right) + \left(\mathbf{R}_{31} \times \mathbf{F}_{31}\right) + \left(\mathbf{R}_p \times \mathbf{P}\right) + \left(\mathbf{R}_d \times \mathbf{F}_{sheath}\right) = 0$$

Expanding cross products in the moment equation gives the moment magnitude as

$$\sum M_z = M_h + \left(R_{21x}F_{21y} - R_{21y}F_{21x}\right) + \left(R_{b1x}F_{b1y} - R_{b1y}F_{b1x}\right) + \left(R_{31x}F_{31y} - R_{31y}F_{31x}\right) \qquad (f)$$
$$+ \left(R_{Px}P_y - R_{Py}P_x\right) + \left(R_{dx}F_{sheath_y} - R_{dy}F_{sheath_x}\right) = 0$$

5 The total of unknowns at this point (including those listed in step 2 above) is 21: F_{b1x}, F_{b1y}, F_{12x}, F_{12y}, F_{21x}, F_{21y}, F_{32x}, F_{32y}, F_{23x}, F_{23y}, F_{13x}, F_{13y}, F_{31x}, F_{31y}, F_{cable_x}, F_{cable_y}, F_{sheath_x}, F_{sheath_y}, P_x, P_y, and M_h. We have only nine equations so far, three in equation set (*a*), one in set (*c*), two in set (*d*) and three in set (*e*). We need twelve more equations to solve this system. We can get seven of them from the Newton's third-law relationships between contacting elements:

$$\begin{aligned} F_{23x} &= -F_{32\,x} & F_{23y} &= -F_{32\,y} \\ F_{21x} &= -F_{12\,x} & F_{21y} &= -F_{12\,y} \\ F_{31x} &= -F_{13x} & F_{31y} &= -F_{13y} \end{aligned} \qquad (g)$$
$$F_{sheath_x} = -F_{cable_x}$$

Table 3–2 - part 1 repeated
Case Study 1A
Given Data

Variable	Value	Unit
F_{13x}	0.0	N
F_{b2x}	0.0	N
F_{b2y}	-267.0	N
θ	184.0	deg
ϕ	180.0	deg
R_{b2x}	39.39	mm
R_{b2y}	2.07	mm
R_{32x}	-50.91	mm
R_{32y}	4.66	mm
R_{12x}	-47.91	mm
R_{12y}	-7.34	mm
R_{21x}	7.0	mm
R_{21y}	19.0	mm
R_{b1x}	47.5	mm
R_{b1y}	-14.0	mm
R_{31x}	-27.0	mm
R_{31y}	30.0	mm
R_{px}	-27.0	mm
R_{py}	0.0	mm
R_{dx}	-41.0	mm
R_{dy}	27.0	mm

3

Two more equations come from the assumption (shown in Figure 3-1) that the two forces provided by the hand on the brake lever and handgrip are equal and opposite:*

$$
\begin{aligned}
F_{b1x} &= -F_{b2x} \\
F_{b1y} &= -F_{b2y}
\end{aligned}
\tag{h}
$$

The remaining three equations come from the given geometry and the assumptions made about the system. The direction of the forces F_{cable} and F_{sheath} are known to be in the same direction as that end of the cable. In the figure it is seen to be horizontal, so we can set

$$
F_{cable_y} = 0; \qquad F_{sheath_y} = 0 \tag{i}
$$

Because of our no-friction assumption, the force F_{31} can be assumed to be normal to the surface of contact between the cable and the hole in part 1. This surface is horizontal in this example, so F_{31} is vertical and

$$
F_{31x} = 0 \tag{j}
$$

6 This completes the set of 21 equations (equation sets *a, c, d, e, g, h, i,* and *j*), and they can be solved for the 21 unknowns simultaneously "as is," that is, all 21 equations could be put into matrix form and solved with a matrix-reduction computer program. However, the problem can be simplified by manually substituting equations *c, g, h, i,* and *j* into the others to reduce them to a set of eight equations in eight unknowns. The known or given data are as shown in Table 3-2, part 1.

7 As a first step, for link 2, substitute equations *b* and *c* in equation *a* to get:

$$
\begin{aligned}
F_{12x} + F_{b2x} + F_{32x} &= 0 \\
F_{12y} + F_{b2y} + F_{32x}\tan\theta &= 0 \\
\left(R_{12x}F_{12y} - R_{12y}F_{12x}\right) + \left(R_{b2x}F_{b2y} - R_{b2y}F_{b2x}\right) + \left(R_{32x}F_{32x}\tan\theta - R_{32y}F_{32x}\right) &= 0
\end{aligned}
\tag{k}
$$

8 Next, take equations *d* for link 3 and substitute equation *c* and also $-F_{32x}$ for F_{23x}, and $-F_{32y}$ for F_{23y} from equation *g* to eliminate those variables.

$$
\begin{aligned}
F_{cable_x} + F_{13x} - F_{32x} &= 0 \\
F_{cable_y} + F_{13y} - F_{32x}\tan\theta &= 0
\end{aligned}
\tag{l}
$$

9 For link 1, substitute equation *f* in *e* and replace F_{21x} with $-F_{12x}$, F_{21y} with $-F_{12y}$, F_{31x} with $-F_{13x}$, F_{31y} with $-F_{13y}$, and F_{sheath_x} with $-F_{cable_x}$ from equation *g*,

$$
\begin{aligned}
-F_{12x} + F_{b1x} - F_{13x} + P_x - F_{cable_x} &= 0 \\
-F_{12y} + F_{b1y} - F_{32x}\tan\theta + P_y &= 0 \\
M_h + \left(-R_{21x}F_{12y} + R_{21y}F_{12x}\right) + \left(R_{b1x}F_{b1y} - R_{b1y}F_{b1x}\right) & \\
+\left(-R_{31x}F_{13y} + R_{31y}F_{13x}\right) + \left(R_{Px}P_y - R_{Py}P_x\right) + R_{dy}\boldsymbol{F}_{cable_x} &= 0
\end{aligned}
\tag{m}
$$

10 Finally, substitute equations *h, i,* and *j* into equations *k, l,* and *m* to yield the following set of eight simultaneous equations in the eight remaining unknowns: F_{12x}, F_{12y}, F_{32x} , F_{13y}, F_{cable_x}, P_x, P_y, and M_h. Put them in the standard form which has all unknown terms on the left and all known terms to the right of the equal signs.

* But not necessarily colinear.